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**Solution to**  
**Eighth International Mathematics Assessment for Schools**  
**Round 1 of Middle Primary Division**

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1. What is the simplified value of

$$13+12+11+\dots+3+2+1+2+3+\dots+11+12+13?$$

( A ) 181      ( B ) 182      ( C ) 183      ( D ) 184      ( E ) 185

【Suggested Solution 1】

$$\begin{aligned} 13+12+11+\dots+3+2+1+2+3+\dots+11+12+13 &= 2 \times (1+2+3+\dots+13) - 1 \\ &= 2 \times \frac{(1+13) \times 13}{2} - 1 \\ &= 14 \times 13 - 1 \\ &= 182 - 1 = 181 \end{aligned}$$

Therefore, the answer is (A).

【Suggested Solution 2】

$$\begin{aligned} 13+12+11+\dots+3+2+1+2+3+\dots+11+12+13 &= (1+2+3+\dots+13) + (2+3+4+\dots+13) \\ &= \frac{(1+13) \times 13}{2} + \frac{(2+13) \times 12}{2} \\ &= \frac{14 \times 13}{2} + \frac{15 \times 12}{2} \\ &= 91 + 90 = 181 \end{aligned}$$

Therefore, the answer is (A).

Answer : ( A )

2. If  $(\Delta \times 2 + 1) \times 3 = 2019$ , then what is the value of  $\Delta$ ?

( A ) 335      ( B ) 336      ( C ) 337      ( D ) 3028      ( E ) 3029

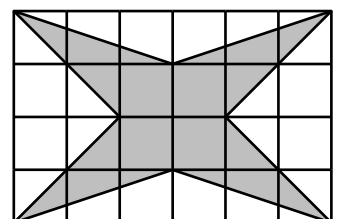
【Suggested Solution】

$$\begin{aligned} (\Delta \times 2 + 1) \times 3 &= 2019 \\ \Delta \times 2 + 1 &= 2019 \div 3 = 673 \\ \Delta \times 2 &= 673 - 1 = 672 \\ \Delta &= 672 \div 2 = 336 \end{aligned}$$

Therefore, the answer is (B).

Answer : ( B )

3. In the figure below, each unit square has side length of 1 cm. What is the area, in  $\text{cm}^2$ , of the shaded region?



( A ) 6

( B ) 8

( C ) 10

( D ) 12

( E ) 14

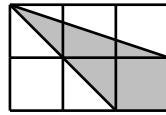
**【Suggested Solution 1】**

In the figure, the area of the 4 unshaded triangles are as follows:  $\frac{1}{2} \times 6 \times 1 = 3 \text{ cm}^2$ ,

$\frac{1}{2} \times 4 \times 2 = 4 \text{ cm}^2$ ,  $\frac{1}{2} \times 6 \times 1 = 3 \text{ cm}^2$  and  $\frac{1}{2} \times 4 \times 2 = 4 \text{ cm}^2$ . Therefore, the area of the shaded area is  $4 \times 6 - 3 - 4 - 3 - 4 = 24 - 14 = 10 \text{ cm}^2$ . Therefore, the answer is (C).

**【Suggested Solution 2】**

Observe that the original figure is composed of four  $2 \times 3$  rectangles as shown below, which are flipped left, flipped right, flipped upside down and rotated  $180^\circ$ :



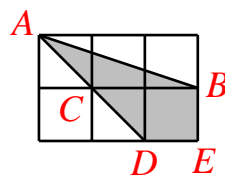
In each of these  $2 \times 3$  rectangles, the areas of the two unshaded triangles are

$\frac{1}{2} \times 3 \times 1 = \frac{3}{2} \text{ cm}^2$  and  $\frac{1}{2} \times 2 \times 2 = 2 \text{ cm}^2$ . Therefore, the area of the shaded region of the

original  $4 \times 6$  rectangle is  $4 \times (2 \times 3 - \frac{3}{2} - 2) = 4 \times \frac{5}{2} = 10 \text{ cm}^2$ . Therefore, the answer is (C).

**【Suggested Solution 3】**

Observe that the original figure is composed of four  $2 \times 3$  rectangles as shown below, which are flipped left, flipped right, flipped upside down and rotated  $180^\circ$ :



In each of the  $2 \times 3$  rectangle, the shaded area can be cut into two parts: triangle  $ABC$  and trapezoid  $BCDE$  (as labeled in the figure above).

Now, notice that the total area of the shaded region is  $\frac{1}{2} \times 2 \times 1 + \frac{1}{2} \times (1 + 2) \times 1 = \frac{5}{2} \text{ cm}^2$ , therefore, the area of the shaded region of the original  $4 \times 6$  rectangle is  $4 \times \frac{5}{2} = 10 \text{ cm}^2$ . Therefore, the answer is (C).

**【Suggested Solution 4】**

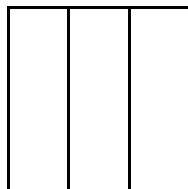
Notice that there are 12 lattice points on the boundary placed on the shaded figure's perimeter, while there are 5 lattice points in the interior which are located inside the

figure. Thus, by applying Pick's Theorem, the area of the shaded region is

$5 + \frac{12}{2} - 1 = 10 \text{ cm}^2$ . Therefore, the answer is (C).

Answer : ( C )

4. A square piece of paper, with a perimeter of 48 cm, is to be cut into three rectangles of the same size as shown in the figure below. What is the perimeter, in cm, of each rectangle that was cut out?



- ( A ) 12      ( B ) 18      ( C ) 24      ( D ) 32      ( E ) 40

**【Suggested Solution】**

It is known that the side length of the square piece of paper is  $48 \div 4 = 12$  cm, therefore, each rectangle has length of 12 cm and width of  $12 \div 3 = 4$  cm, Therefore, the perimeter of each rectangle is  $2 \times (12 + 4) = 32$ . Therefore, the answer is (D).

Answer : ( D )

5. In the equation below, fill in the five numbers 7, 50, 51, 55, and 63 into the five circles to make the equality true. What is the sum of the numbers in the first three circles?

$$\square + \square + \square - \square = \square$$

- ( A ) 105      ( B ) 106      ( C ) 113      ( D ) 114      ( E ) 118

**【Suggested Solution】**

By moving the 4<sup>th</sup> term of the original equation to the other side, the equation can be rewritten as follows:

$$\square + \square + \square = \square + \square$$

Now, we have divided the five numbers into two groups, where on side, we have three numbers and the other side, we have two numbers, and that the sum of the numbers in each group are equal. Notice that  $7 + 50 + 51 + 55 + 63 = 226$ , therefore, the sum of the numbers in each side of the equation should be  $226 \div 2 = 113$ . Now, notice that one possible permutation is  $113 = 50 + 63 = 7 + 51 + 55$ , thus, the sum of the numbers in the first three circles is 113. Therefore, the answer is (C).

Answer : ( C )

6. How many different 3-digit numbers are there such that the sum of its digits is equal to 4?

- ( A ) 7      ( B ) 8      ( C ) 9      ( D ) 10      ( E ) 11

**【Suggested Solution 1】**

Observe that if 4 is written as a sum of up to three single digits, it can be one of the

following combinations:  $4=1+3=2+2=1+1+2$ , therefore, the number of 3-digit numbers with sum of digits equals 4 are as follows: 103, 112, 121, 130, 202, 211, 220, 301, 310 and 400, for a total of 10. Therefore, the answer is (D).

**【Suggested Solution 2】**

Observe that if 4 is written as a sum of up to three digits, it can be one of the following combinations:  $4=1+3=2+2=1+1+2$ ,  
Therefore, the number of 3-digit numbers with sum of digits equal to 4 can be counted in the following 4 cases:

**Case 1:** If the 3-digit number consists of the digits 0, 0 and 4: there are 3 such numbers, but 004 and 040 are not 3-digit numbers, so only 1 number satisfies the condition.

**Case 2:** If the 3-digit number consists of the digits 0, 1 and 3: there are a total of  $3 \times 2 \times 1 = 6$  such numbers, but 013 and 031 are not 3-digit numbers, so there are 4 numbers that satisfies the condition.

**Case 3:** If the 3-digit number consists of the digits 0, 2 and 2: there are a total of 3 such numbers, but 022 is not a three-digit number, so there are 2 numbers that satisfies the condition.

**Case 4:** If the 3-digit number consists of the digits 1, 1 and 2, then there are 3 numbers that satisfies the condition.

So, there are a total of  $1+4+2+3=10$  numbers that satisfy the condition of the problem. Therefore, the answer is (D).

Answer : ( D )

7. There are a total 80 pandas in a zoo, all of which are housed in three different halls. The number of pandas in Hall 1 is twice that in Hall 2, and the number of pandas in Hall 2 is three times that in Hall 3. How many pandas are there in Hall 2?

( A ) 12      ( B ) 24      ( C ) 36      ( D ) 48      ( E ) 60

**【Suggested Solution】**

The number of pandas in Hall 1 is twice that of Hall 2, and the number of pandas in Hall 2 is three times that of Hall 3. Therefore, the number of pandas in Hall 1 is

$2 \times 3 = 6$  times that of Hall 3. So the total number of pandas in Hall 3 is  $\frac{80}{1+3+6} = 8$

pandas, therefore, the total number of pandas in Hall 2 is  $8 \times 3 = 24$ . Therefore, the answer is (B).

Answer : ( B )

8. Every page of a book is either all text or all illustrations and that there are exactly three pages of illustrations between every two pages of text, that is, if the last page of the book is text only, then one page before and after the three-page illustration are all-texts. If the last page is an illustration, in addition to the illustrations on the last few pages, one page before and after the other three pages of illustrations is text. If the first page of the book is text, and it has a total of 136 pages, how many pages are there in this book are illustrations only?

( A ) 34      ( B ) 81      ( C ) 102      ( D ) 108      ( E ) 120

**【Suggested Solution】**

It can be seen that except for the case where the last page is a text, every page of text-only is followed by a three-page of illustration-only pages and the last text page may be a 3, 2 or 1 illustration-only page. Now since 136 is a multiple of 4, it can be judged that the last text page is followed by a 3-page illustration. Thus, the number of illustration-only pages is  $136 \times \frac{3}{4} = 102$ . Therefore, the answer is (C).

Answer : ( C )

9. For any positive integer, we compute by following the operating rules: If the number is an even number, then we divide it by 2; and if it is odd, then we add 1 to it. We continue to do these operations until we arrive with the value of “1” for the first time. How many numbers will leave a value of “1” after four operations?

( A ) 2      ( B ) 3      ( C ) 4      ( D ) 5      ( E ) 6

**【Suggested Solution】**

The number obtained after the odd number is increased by one is always an even number, and the number obtained by dividing the even number by 2 may be an odd number or an even number, so it can be either of the two scenarios:

- If the number obtained after the operation is an odd number, the number before the operation is the odd number multiplied by 2;
- If the number obtained after the operation is even, the number before the operation is the even number multiplied by 2 or minus 1.

Since we obtain “1” after four operations which is an odd number, the number obtained after three operations must be  $1 \times 2 = 2$ .

Since we obtain “2” after three operations, which is an even number, the number obtained after the second operation can be  $2 \times 2 = 4$  or  $2 - 1 = 1$ , but the operation is stopped if we get 1, so, the number obtained after the second operation must be 4.

Since we obtain “4” after two operations, which is an even number, the number obtained after one operation can be  $4 \times 2 = 8$  or  $4 - 1 = 3$ ,

- (i) if an even number of 8 is obtained after an operation, the original number can be  $8 \times 2 = 16$  or  $8 - 1 = 7$ ,
- (ii) if an odd number of 3 is obtained after one operation, the original number is  $3 \times 2 = 6$ .

Therefore, it is known that a total of 3 numbers that can get a value of “1” after four operations. Therefore, the answer is (B).

Answer : ( B )

Note: There is one solution if we go one operation, which is 2; There is also one solution if we do two operations, which is 4; There are two solutions if we do three operations, which are 3 and 8; and there are three solutions if we do four operations, which are 6, 7, and 16. The answer to this question is essentially a Fibonacci

sequence.

10. Among the numbers from 0 to 200, what is the average of all numbers that are multiples of 3?

( A ) 95      ( B ) 96      ( C ) 99      ( D ) 100.5      ( E ) 101

【Suggested Solution 1】

From the numbers from 0 to 200, the numbers that are multiple of 3 are as follows: 0, 3, 6, ..., 198. Now, we do pairing of the following numbers: (0, 198), (3, 195), ..., (96, 102), and we have an extra 99 which has no pair, from this, the average of every pairwise is 99, so the average is also 99. So, the answer is (C).

【Suggested Solution 2】

It can be seen that among all the numbers from 0 to 200, the multiple of 3 is 0, 3, 6, ..., 198. So, in total we have  $\frac{198}{3} + 1 = 67$  terms. Therefore, the average of these numbers

is  $\frac{(0+198) \times 67}{2} = \frac{99 \times 67}{67} = 99$ . Therefore, the answer is (C).

Answer : ( C )

11. What is the tens digit of the simplified value of the following expression:  
 $1+12+123+1234+12345+123456+1234567+12345678+123456789$ ?

( A ) 0      ( B ) 2      ( C ) 4      ( D ) 6      ( E ) 8

【Suggested Solution】

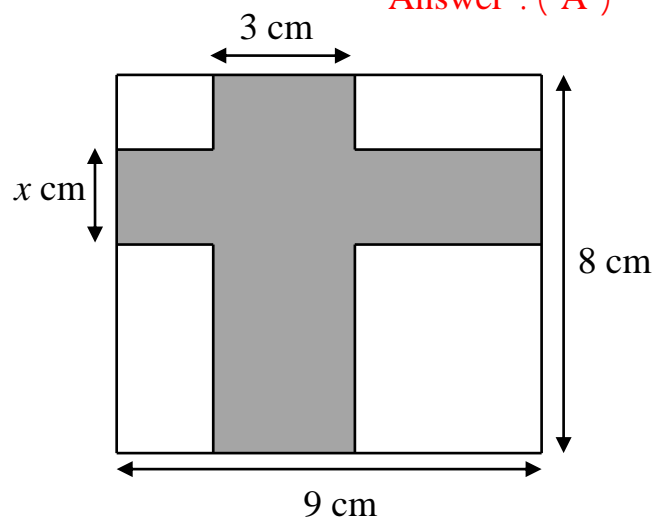
It can be observed that it is only necessary to consider adding the last two digits of each number in the equation to get the last 2 digits.

Since  $1+12+23+34+45+56+67+78+89=405$ , then the tens digit is 0.

Therefore, the answer is (A).

Answer : ( A )

12. In the figure below, the largest rectangle has dimensions 9 cm long by 8 cm wide, with a cross-section formed by two shaded rectangles in the middle. If the total area of the shaded parts is exactly equal to the area of the un-shaded parts, then what is the value, in cm, of  $x$ ?



- ( A ) 1                      ( B ) 2                      ( C ) 3                      ( D ) 4                      ( E ) 5

【Suggested Solution 1】

It can be seen that the area of the largest rectangle is  $9 \times 8 = 72 \text{ cm}^2$ , and because the total area of the shaded portions is exactly equal to the area of the un-shaded portions, then their area should be  $72 \div 2 = 36 \text{ cm}^2$ , thus,  $3 \times 8 + x \times (9 - 3) = 36$ , which yields  $x = 2$ . Therefore, the answer is (B).

【Suggested Solution 2】

It can be seen that the dimensions of the un-shaded portion have length  $9 - 3 = 6 \text{ cm}$  and with  $8 - x \text{ cm}$ . and since the area of the largest rectangle is  $9 \times 8 = 72 \text{ cm}^2$  and because the total area of the shaded portions is exactly equal to the area of the un-shaded portions, then their area should be  $72 \div 2 = 36 \text{ cm}^2$ , you have the equation  $6(8 - x) = 36$ . Solving for  $x$ , which will yield  $x = 2$ . Therefore, the answer is (B).

Answer : ( B )

13. Use the digits 2, 0, 1, and 9 exactly once to form different 4-digit numbers.  
How many of these 4-digits numbers will be greater than 1905?

- ( A ) 12                      ( B ) 13                      ( C ) 15                      ( D ) 17                      ( E ) 19

【Suggested Solution】

**Case 1:** If the thousands digit is 1, there is one number (1920) that is greater than 1905.

**Case 2:** If the thousands digit is 2 or 9, then, the four-digit number is always greater than 1905. If we have already selected the thousand digits, then there are 3 ways in choosing for the hundreds digit, then there are 2 ways in choosing for the tens digit, and there is only 1 way in choosing for the units digit. Therefore, there will be a total of  $2 \times 3 \times 2 \times 1 = 12$  numbers that will be greater than 1905 for this case.

So, there are a total of  $1 + 12 = 13$  numbers that will be greater than 1905. Therefore, the answer is (B).

Answer : ( B )

14. After deleting one of the six numbers 2, 3, 12, 26, 29 and 41, the five remaining numbers can be divided into two groups with the same sum. What is the number that was deleted?

- ( A ) 3                      ( B ) 12                      ( C ) 26                      ( D ) 29                      ( E ) 41

【Suggested Solution】

Notice that  $2 + 3 + 12 + 26 + 29 + 41 = 113$ , so the sum of the five numbers left must be a multiple of 2, so we can deduce that the number deleted must be an odd number.

So we have 3 different cases:

**Case 1:** If the number deleted is 41, then the sum of the five numbers left is  $2 + 3 + 12 + 26 + 29 = 72$ . So, the sum of the numbers in each group must be  $72 \div 2 = 36$ . So, the sum of the remaining numbers in the group that contains 29 is 7, and on the remaining numbers, the sum of the numbers less than 7 is  $2 + 3 = 5 \neq 7$ , so it's impossible.

**Case 2:** If the number deleted is 29, then the sum of the five numbers left is  $2 + 3 + 12 + 26 + 41 = 84$ . So, the sum of the numbers in each group must be  $84 \div 2 = 42$ . So, the sum of the remaining numbers in the group that contains 41 must be 1, but we don't have 1, so it's impossible.

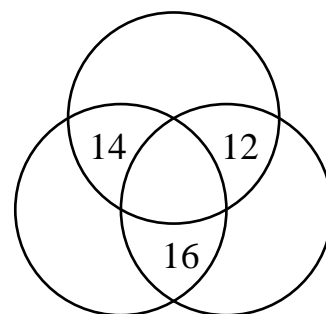
**Case 3:** If the number deleted is 3, then the sum of the five numbers left is  $2 + 12 + 26 + 29 + 41 = 110$ . So, the sum of the numbers in each group must be  $110 \div 2 = 55$ . So, the sum of the remaining numbers in the group that contains 41 must be 14, out of the five numbers, we have  $2 + 12 = 14$ . Therefore, the other group must be  $26 + 29 = 55$ , that is, one group contains the numbers 41, 12 and 2 and the other group contains the numbers 26 and 29.

Thus, the number deleted is 3. Therefore, the answer is (A).

Answer : ( A )

15. As shown in the diagram, three of the seven areas enclosed by the three circles are each filled with a number, and it is known that the numbers 13, 15, 17, and 18 can be filled inside the four blank areas such that the sum of the four numbers in each circle are all equal. What is the sum?

- ( A ) 58      ( B ) 59      ( C ) 60  
( D ) 61      ( E ) 62



**【Suggested Solution】**

Notice that there are 3 odd numbers and 1 even number in the 4 numbers: 13, 15, 17, and 18, and that the already filled-in numbers 12, 14, and 16 are all even numbers, then, the only even number 18 must be filled-in to the center to make the sum of the four numbers in each circle equal. Be sure to fill in the middle area so that the sum of the four numbers in each circle is odd, otherwise the sum of the four numbers in the circle where 18 is located is odd and the sum of the four numbers in the other two circles it is even. (See Figure 1).

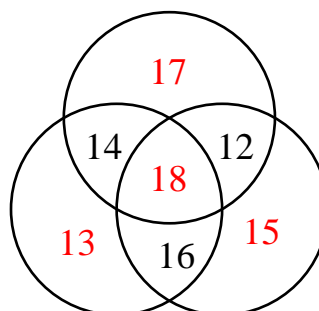
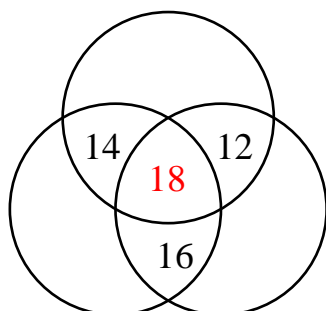




Figure 1

Figure 2

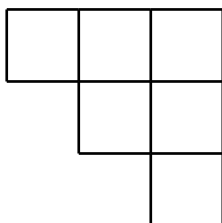
It can be seen that the sum of the numbers in the three circles at this time from largest to smallest are as follows:  $14+16+18=48$ ,  $12+16+18=46$  and  $12+14+18=44$ , thus, we fill in the numbers 13, 15 and 17, to the corresponding blank areas in the circle sequentially as well to satisfy the conditions (See Figure 2). That is,

$$13+14+16+18=12+15+16+18=12+14+17+18=61.$$

Therefore, the answer is (D).

Answer : ( D )

16. In the figure below, each unit square has a side length of 1 cm. How many squares and rectangles are there?



- ( A ) 9                      ( B ) 12                      ( C ) 14                      ( D ) 15                      ( E ) 16

### 【Suggested Solution】

It can be seen that the side length of each unit square in the figure is an integer, so we can use the area to count for the rectangles. It can be seen that the area of this figure is  $6\text{ cm}^2$  and the pattern is not rectangular, so we consider the following 5 cases:

**Case 1:** The rectangle with an area of 1 square unit must be of dimension  $1\times 1$ , so we have 6.

**Case 2:** The rectangle with an area of 2 square units must be of dimension  $1\times 2$ , so we have 6.

**Case 3:** The rectangle with an area of 3 square units must be of dimension  $1\times 3$ , so we have 2.

**Case 4:** The rectangle with an area of 4 square units must be of dimension  $2\times 2$ , so we have 1.

**Case 5:** The rectangle with an area of 4 square units must be of dimension  $1\times 5$ , but we have no such rectangles.

So, in total, we have a total of  $6+6+2+1=15$  rectangles. Therefore, the answer is (D).

Answer : ( D )

17. A theater has nine rows of seats. The first row has a total of 19 seats and starting from the second row, each row has two more seats than the previous row. How many seats does this theater have in total?

- ( A ) 173      ( B ) 187      ( C ) 243      ( D ) 261      ( E ) 280

【Suggested Solution】

It can be seen that the number of seats in the nine rows of seats are as follows: 19, 21, 23, 25, 27, 29, 31, 33 and 35 in order (from front to back), so the theater has a total of  $19 + 21 + 23 + 25 + 27 + 29 + 31 + 33 + 35 = \frac{(19 + 35) \times 9}{2} = 27 \times 9 = 243$  number of seats.

Therefore, the answer is (C).

Answer : ( C )

18. Divide the six positive integers 2, 3, 7, 13, 18, and  $X$  into three groups so that the first group contains one number, the second group contain two numbers, and the third group contain three numbers, where  $X$  must belong in the third group. If the number in the first group is greater than the sum of the numbers in the second group, and that the sum of the numbers in the second group is greater than the sum of the numbers in the third group, then what is the maximum possible value of  $X$ ?

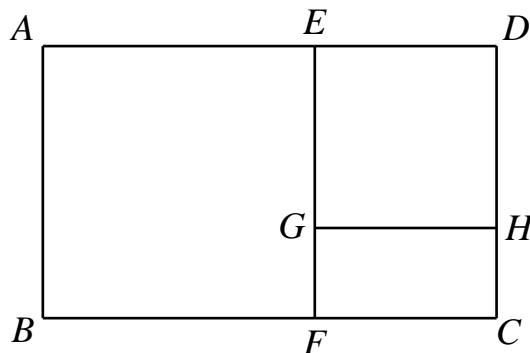
- ( A ) 4      ( B ) 5      ( C ) 6      ( D ) 7      ( E ) 8

【Suggested Solution】

It can be seen that the number in the first group is greater than the sum of the numbers in the second group, and is also greater than the sum of the number of the third group, so therefore, the number in the first group must be the maximum of the six numbers. Then because the unknown number  $X$  is in the third group, therefore, the number in the first group must be 18. Now, the sum of the two numbers in the second group must be less than 18. At this time, we check the different possible scenarios:  $13 + 7 = 20 > 18 > 13 + 3 = 16$ , thus, in order to maximize the value of  $X$  in the third group, the two numbers of the second group must be taken as 13 and 3. Now, the sum of the three numbers of the third group is  $2 + 7 + X = 9 + X < 16$ , therefore, the largest possible value of  $X$  is 6. Therefore, the answer is (C).

Answer : ( C )

19. In the figure below, rectangle  $ABCD$  has a perimeter of 32 cm, where both  $ABFE$  and  $EGHD$  are squares. If the perimeter of  $GFCH$  is 12 cm, then what is the area, in  $\text{cm}^2$ , of rectangle  $ABCD$ ?



- ( A ) 28      ( B ) 39      ( C ) 48      ( D ) 55      ( E ) 60

【Suggested Solution 1】

Since the perimeter of the rectangle  $ABCD$  is 32 cm, so  $BC + CD = 16$  cm and since the perimeter of rectangle  $GFCH$  is 12 cm, so  $FC + CH = 6$  cm. Since it is also known that  $ABFE$  and  $EGHD$  are squares, then  $AE = BF$  and  $ED = DH$ , thus,

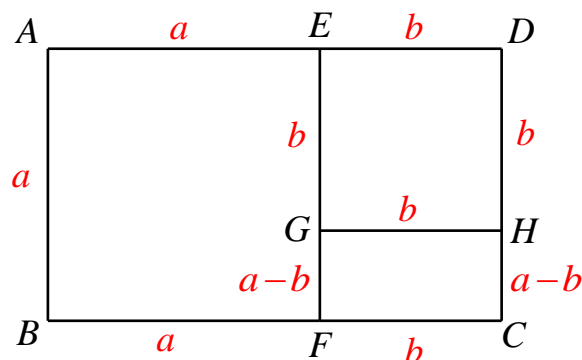
$$AD = AE + ED = BF + DH = (BC - FC) + (CD - CH) = 16 - 6 = 10 \text{ cm},$$

Thus,  $CD = 16 - BC = 16 - 10 = 6$  cm, therefore, the area of rectangle  $ABCD$  is  $6 \times 10 = 60 \text{ cm}^2$ . Therefore, the answer is (E).

【Suggested Solution 2】

Let the side length of the square  $ABFE$  is  $a$  cm, and let the side length of the square  $EGHD$  is  $b$  cm. It is also known that  $FC = GH = ED = b$ ,

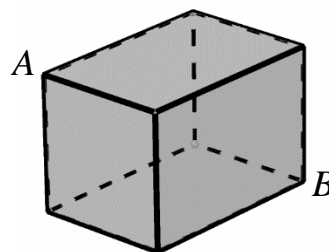
$GF = CH = CD - HD = AB - HD = a - b$  and the length of each line segment is shown in the diagram below:



Now, the perimeter of the rectangle  $GFCH$  is  $(a - b) + b + (a - b) + b = 2a = 12$  cm, so from this,  $a = 6$  and the perimeter of rectangle  $ABCD$  is  $(a + b) + a + (a + b) + a = 4a + 2b = 24 + 2b = 32$  cm, so  $b = 4$ , so from this, the length of rectangle  $ABCD$  is  $6 + 4 = 10$  and width is 6 cm, therefore, the area of rectangle  $ABCD$  is  $6 \times 10 = 60 \text{ cm}^2$ . Therefore, the answer is (E).

Answer : ( E )

20. As shown in the figure below, an ant starting from vertex  $A$  of the cuboid needs to move along the edges to reach its destination, vertex  $B$ . If the ant can only pass through three edges, how many possible paths can the ant crawl to reach the destination?



- ( A ) 3      ( B ) 6      ( C ) 9      ( D ) 12      ( E ) 15

【Suggested Solution】

The ant have a total of 3 edges starting from  $A$ , and after selecting the edge of the first one, the second edge has two choices and the third edge is also determined. So, there is a total of  $3 \times 2 = 6$  different ways that the ant can travel from point  $A$  to  $B$ .

Therefore, the answer is (B).

Answer : ( B )

21. A 5-digit number is formed by using digits that are all different. It is known that four of the digits used are 3, 6, 8 and 9, and that the difference between any two adjacent digits is greater than 3. How many 5-digit numbers are there that satisfy the condition?

【Suggested Solution】

From the condition, it can be seen that the three digits 6, 8, and 9 cannot be adjacent. That is, these three numbers must be on the ten thousands, hundreds and units digit place. Now, notice that the difference between the digits 6 and 3 is exactly 3, therefore, the digit 3 must be always between digits 8 and 9 and 6 cannot be a in the hundreds place, otherwise, the digit 6 must be adjacent to digit 3. Because of this, there are only two choices for hundreds place, and when hundreds place is already chosen, there are two choices for the tens place and the units place are also determined. Therefore, we have  $2 \times 2 = 4$  ways, i.e., for the numbers in the form  $\overline{6X839}$ 、 $\overline{6X938}$ 、 $\overline{839X6}$ 、 $\overline{938X6}$  · X can only be located between 6 and 8, so it can only take values 0, 1, 2, That is, there are a total of  $4 \times 3 = 12$  five-digit numbers that satisfies the condition.

Answer : 012

22. Joe has a total of 31 dollars that he will use to purchase four different items that have unit prices of 2, 3, 5 and 7 dollars each respectively. If he needs to purchase at least one piece of each item, and Joe has to use all his money without leftovers, in how many different ways can he buy the goods?

【Suggested Solution】

Since at least one item must be purchased for each item, when Joe buys one item for each item, he will have a total of  $31 - 7 - 5 - 3 - 2 = 14$  dollars left. At this time, each product does not necessarily need to be purchased again. Then by enumerating all possible scenarios, we have the following:

$$\begin{aligned} 14 &= 7 \times 2 \\ &= 7 + 5 + 2 \\ &= 7 + 3 + 2 \times 2 \\ &= 5 \times 2 + 2 \times 2 \\ &= 5 + 3 \times 3 \\ &= 5 + 3 + 2 \times 3 \\ &= 3 \times 4 + 2 \\ &= 3 \times 2 + 2 \times 4 \\ &= 2 \times 7 \end{aligned}$$

So, there are a total of 9 different ways.

Answer : 009

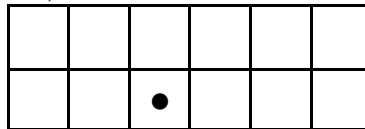
23. It is known that the sum of the ages of A and B this year is 60 years old. 2 years ago, the sum of the ages of B and C is also 60 years old and 3 years from now, the sum of the ages of A and C is still 60 years old. How old is A this year?

【Suggested Solution】

Because the sum of the ages of B and C two years ago is 60 years old, then, the age of B and C now is  $60 + 2 + 2 = 64$  years old. And 3 years from now, the sum of ages of A and C is also 60 years old, so the sum of ages of A and C now is  $60 - 3 - 3 = 54$  years old. So, we know that the sum of the current ages of the three persons is  $(60 + 64 + 54) \div 2 = 89$  years old. So, this year, the age of A is  $89 - 64 = 25$  years old.

Answer : 025

24. A coin is placed in one of the unit squares in the grid as shown below. How many rectangles (including squares) can be formed that contains the coin?



【Suggested Solution 1】

It can be seen that the length of each side of each unit square in the figure is an integer unit, so their area can be used to classify the rectangles. It can be seen that the area of this square table is 12 square units, so the following can be obtained:

The rectangle with an area of 1 square unit must be of dimension  $1 \times 1$ , so we have 1.  
The rectangle with an area of 2 square units must be of dimension  $1 \times 2$ , so we have 3.  
The rectangle with an area of 3 square units must be of dimension  $1 \times 3$ , so we have 3.  
The rectangle with an area of 4 square units, it can be as follows:

For dimensions  $1 \times 4$ , so we have 3.

For dimensions  $2 \times 2$ , so we have 2.

The rectangle with an area of 5 square units must be of dimension  $1 \times 5$ , so we have 2.

The rectangle with an area of 6 square units, it can be as follows:

For dimensions  $1 \times 6$ , so we have 1.

For dimensions  $2 \times 3$ , so we have 3.

The rectangle with an area of 8 square units must be of dimension  $2 \times 4$ , so we have 3.

The rectangle with an area of 10 square units must be of dimension  $2 \times 5$ , so we have 2.

The rectangle with an area of 12 square units must be of dimension  $2 \times 6$ , so we have 2.

So, we have a total of  $1 + 3 + 3 + 3 + 2 + 2 + 1 + 3 + 3 + 2 + 1 = 24$  different rectangles (including squares) that can be formed that includes the coin.

【Suggested Solution 2】

Notice that there are 3 horizontal lines and 7 vertical lines in the grid, and each rectangle formed must be bordered by two horizontal lines and two vertical lines. The rectangle containing the coin must always have the bottom-most horizontal line, therefore, there are two ways to choose the other horizontal line. Moreover, on the

three vertical lines on the left side of the coin, we must choose one, and on the four vertical lines on the right side of the coin, we must also choose one, so there are  $3 \times 4 = 12$  ways in choosing the vertical lines. So, there is a total of  $(1 \times 2) \times (3 \times 4) = 24$  different rectangles (including squares) that can be formed that includes the coin.

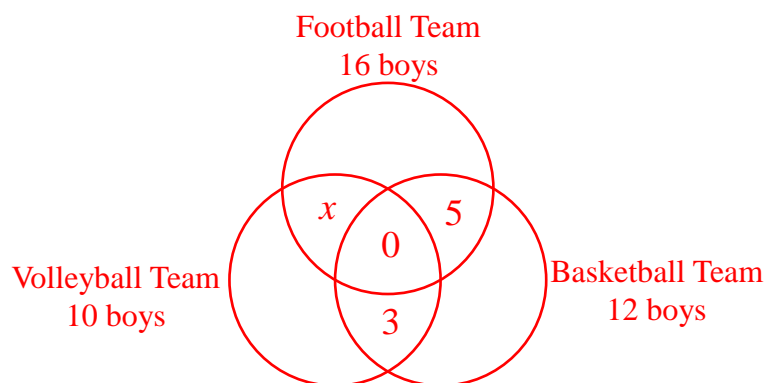
Answer : 024

25. There are 25 boys in a class, 16 of whom are members of the football team, 12 in the basketball team and 10 in the volleyball team. It is known that no one participates in all three sports at the same time, and each boy participates in at least one team. If five boys participate in both the football and basketball teams, and three boys participate in both the basketball and volleyball teams. How many boys are there that participate in both the football and volleyball teams?

【Suggested Solution 1】

Suppose there are  $x$  boys who participate in both the football and volleyball teams.

The following Venn Diagram represents the given problem:



Since  $16 + 12 + 10 - 5 - 3 - x = 25$ , then  $x = 5$ .

【Suggested Solution 2】

Since we know that the number of boys playing for only the basketball team is  $12 - 5 - 3 = 4$  boy, therefore,  $25 - 4 = 21$  boys participate in the football team or the volleyball team. It is known that the total number of boys in the football and volleyball teams is  $16 + 10 = 26$  boys, but we can see that  $26 - 21 = 5$  boys has been counted twice, thus, a total of 5 boys participate in both the football team and the volleyball team.

Answer : 005