International Mathematics Assessments for Schools

2018 JUNIOR DIVISION FIRST ROUND PAPER

Time allowed：75 minutes

When your teacher gives the signal, begin working on the problems.

INSTRUCTION AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. No calculators, slide rules, log tables, math stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 20 multiple-choice questions, each with 5 choices. Choose the most reasonable answer. The last 5 questions require whole number answers between 000 and 999 inclusive. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a mathematics assessment, not a test; do not expect to answer all questions.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.

THE ANSWER SHEET

1. Use only pencils.
2. Record your answers on the reverse side of the Answer Sheet (not on the question paper) by FULLY filling in the circles which correspond to your choices.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places. So please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The IMAS reserves the right to re-examine students before deciding whether to grant official status to their scores.

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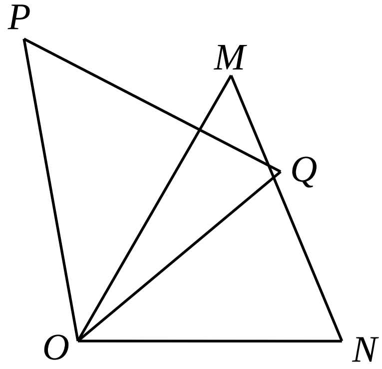
Questions 1-10, 3 marks each

1. Calculate the value of the expression: $2020^2 - 2019^2 - \sqrt{(-2018)^2}$.

(A) 2021 (B) 2022 (C) 2037 (D) 4039 (E) 6057

2. In the figure below, it is known that $\triangle POQ \cong \triangle MON$, $\angle PON = 100°$ and $\angle MOQ = 20°$. What is the measure, in degrees, of $\angle POQ$?



(A) 20 (B) 30 (C) 40 (D) 45 (E) 60

3. If $x = 2$ and $y = 3$, then what is the value of $x^4 + y^4 - x^3 - y^3 + x^2 + y^2$?

(A) 71 (B) 72 (C) 75 (D) 83 (E) 85

4. Two positive integers *m* and *n* satisfy the following conditions: When *m* is divided by 35, it leaves a remainder of 12 and when *n* is divided by 21, it leaves a remainder of 15. What is the remainder when $m - n$ is divided by 7?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

5. If $x^2 - 4x + 4 + \sqrt{xy - 2018} = 0$, then what is the value of *y*?

(A) 0 (B) 1009 (C) 2018 (D) 4036 (E) Uncertain

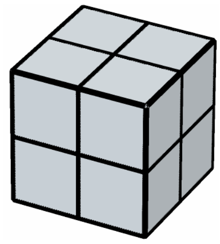
6. If $x = 3$, then what is the value of $\sqrt{x-1+\sqrt{x-1+\sqrt{x-1+\sqrt{x+1}}}}$?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

7. The teacher has 2 identical pens and 3 identical pencils to be given out as prizes to two of his students. If each student should receive at least one object, in how many ways can the teacher distribute the prizes?

(A) 5 (B) 6 (C) 8 (D) 9 (E) 10

8. As shown in the figure below, a $2\times2\times2$ cube is formed by placing together eight $1\times1\times1$ cubes. If one $1\times1\times1$ cube is removed, what will be the surface area of the remaining figure?



（A）24 （B）25 （C）26 （D）27 （E）28

9. If positive integers m and n satisfy that $m^2-n^2=13$, then what is the value of m^2+n^2?

（A）13 （B）36 （C）49 （D）75 （E）85

10. A palindrome number is a positive integer that is the same when read forwards or backwards. The numbers 909 and 1221 are examples of palindromes. How many three-digit palindrome numbers are divisible by 9?

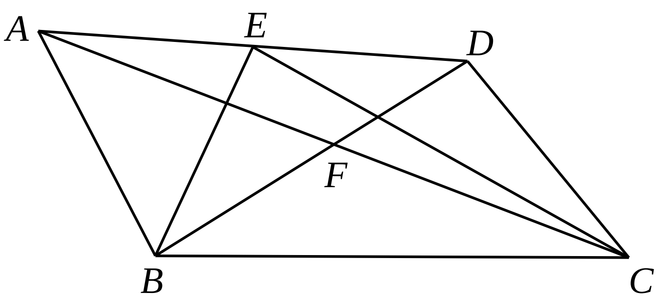
（A）10 （B）12 （C）15 （D）20 （E）24

Questions 11-20, 4 marks each

11. The greatest common divisor of n and 24 is 2, while the greatest common divisor of $n+1$ and 24 is 3. Which of the following numbers cannot be n?

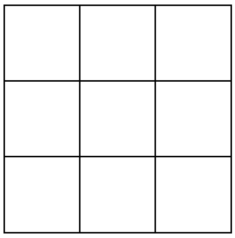
（A）2 （B）14 （C）20 （D）38 （E）50

12. In the figure below, let point E be the midpoint of AD and point F be the midpoint of AC. If the area of triangle ABF is 8 cm^2 and area of ADF is 6 cm^2, then what is the area, in cm^2, of triangle BCE?



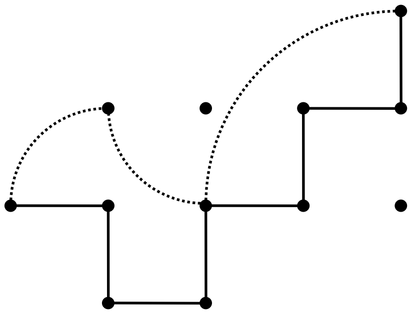
（A）12 （B）13 （C）14 （D）15 （E）16

13. Shade 3 unit squares on the 3×3 grid below, such that there must be two shaded squares in some row and two shaded squares in some column but it must not have three shaded squares in any row or column. Find the total number of ways in shading the figure.



（A） 6　（B） 18　（C） 36　（D） 54　（E） 72

14. In the figure below, the eight line-segments drawn are all equal to 1 m and the three dotted lines are all quarter arcs. What the is difference, in m, between the total length of all the line segments and total length of all the dotted arcs? (Use $\pi=3.14$)



（A） 0.28　（B） 0.72　（C） 1.28　（D） 1.72　（E） 4.86

15. Starting from $\frac{3}{4}$, add 2 to the numerator or add 3 to the denominator for each operation, but not both, and no reduction is performed. At least how many operations one needs to get a fraction again that is of the same value as $\frac{3}{4}$?

（A） 13　（B） 17　（C） 20　（D） 26　（E） 34

16. Let a, b, c and d be consecutive positive integers such that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{36}+\frac{1}{45}=1$. What is the value of $a+b+c+d$?

（A） 10　（B） 12　（C） 14　（D） 16　（E） 18

17. Replace all the 9 variables in the expression $a+\frac{c}{b}+d+\frac{f}{e}+g+\frac{i}{h}$ using the digits 1, 2, 3, ⋯ 9, where each digit is only used once. Find the maximum possible value of the result.

(A) 25 (B) $31\frac{2}{3}$ (C) $33\frac{2}{3}$ (D) $33\frac{5}{6}$ (E) $34\frac{1}{6}$

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18. An ant crawls on the plane. It starts at point A and crawls for 1 cm, then turns $60°$ to the right; crawls for another 2 cm and again turns $60°$ to the right; crawls for another 3 cm and turns $60°$ to the right; crawls for another 4 cm and turns $60°$ to the right; and finally it craws 5 cm and reaches F. What is the distance, in cm, between A and F?

(A) 0 (B) 3 (C) $3\sqrt{3}$ (D) 6 (E) $6\sqrt{3}$

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19. Arrange all proper fractions in a sequence such that the denominators are all in non-decreasing order, and while for equal denominators, the numerator is arranged in increasing order. The resulting sequence is as follows:

$$\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \cdots.$$

It is known that the sum of first n terms of this sequence is an integer, which of the integers below is a possible value for n?

(A) 2015 (B) 2016 (C) 2017 (D) 2018 (E) 2019

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20. Using the digits 1, 2, 3, 4, 5, 6, 7 and 8 only once, create a sequence such that there is one number between 1 and 2, two numbers between 2 and 4, three numbers between 3 and 6 and four numbers between 4 and 8. How many different ways can we do this?

(A) 12 (B) 24 (C) 36 (D) 48 (E) 60

Questions 21-25, 6 marks each

21. An integer is known to be both a multiple of 3 and 7. Among all its divisors, there is one more multiple of 7 than multiple of 3. What is the least possible integer that satisfies the condition?

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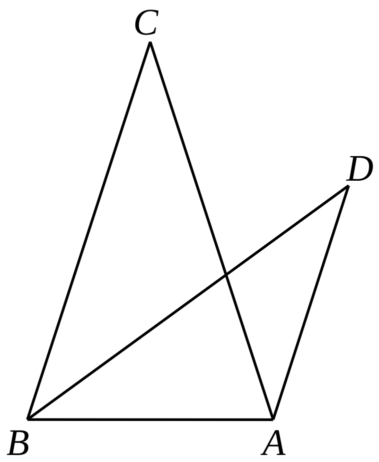
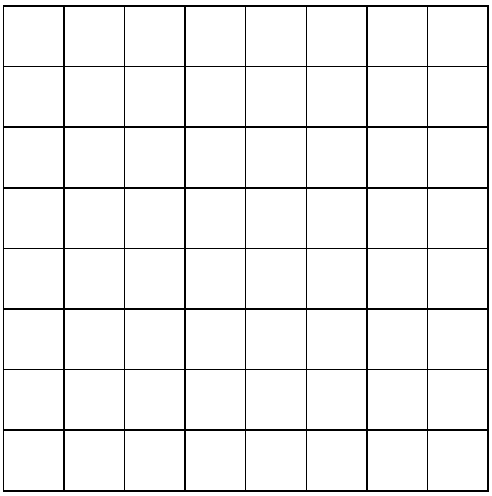
22. In the figure below, it is known that $BC//AD$, $BC = AC$, $BA = AD$ and $\angle C = \angle D$. Find the measure, in degrees, of $\angle BAC$?



23. Let a, b and c be real numbers such that $abc = 1$ and $a + b + c = ab + bc + ca = 6$. What is the value of $a^3 + b^3 + c^3$?

24. Let a be a positive integer such that $2018 - a^2$ is also positive. What is the maximum possible number of divisors of $2018 - a^2$?

25. Cut the 8×8 square table below into rectangles along grid lines such that no two rectangles are identical. What is the maximum number of rectangles one can get? (Note: A square is considered a rectangle.)



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