2012 JUNIOR DIVISION FIRST ROUND SOLUTION

1. What is the value of $2012^0+(-1)^2+|-2012|$?

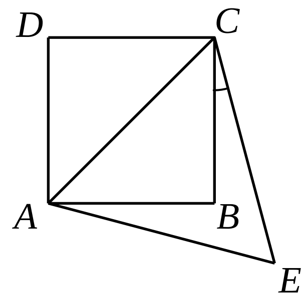
(A) -2010 (B) 1 (C) 2012 (D) 2013 (E) 2014

【Suggested solution】

Since $2012^0=1$、$(-1)^2=1$、$|-2012|=2012$，the value of this algebraic expression is 1+1+2012=2014。

Answer：(E)

2. In the diagram below, $ABCD$ is a square and ACE is an equilateral triangle. What is the measure, in degrees, of $\angle BCE$?



(A) 15 (B) 20 (C) 25
(D) 30 (E) cannot be determined

【Suggested solution】

Since $ABCD$ is a square，ABC is an isosceles right angled triangle and $\angle ACB=45°$. Since ACE is a equilateral triangle, $\angle ACE=60°$, thus $\angle BCE=60°-45°=15°$.

Answer：(A)

3. The smallest interior angle of a triangle is 50°. Which of the following statements about this triangle is correct?

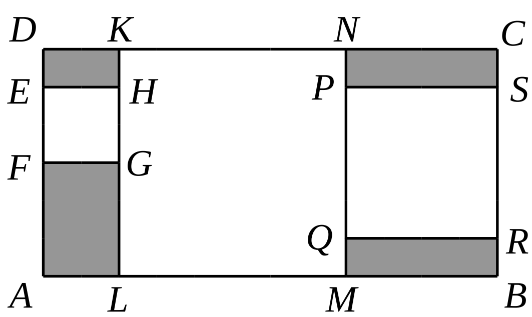
(A) It must be isosceles. (B) It must be right angled.
(C) It must be acute angled. (D) It must be obtuse angled.
(E) None of these is correct.

【Suggested Solution】

Since the smallest interior angle in a triangle is 50°, the maximum interior angle will not exceed 180°－50°－50° = 80° . Therefore, this triangle must be acute angled. It does not have to be isosceles as the angles may be (50°, 60°, 70°).

Answer：(C)

4. The diagram to the below shows three squares $EFGH$, $KLMN$ and $PQRS$ inside a rectangle $ABCD$. The areas of the three squares are 1 cm^2, 9 cm^2 and 4 cm^2 respectively. What is the sum of areas of the shaded regions in cm^2?



(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

【Suggested solution】

The corresponding length of three square is 1 cm, 3 cm, 2 cm, so AB=1+3+2=6 cm.
The area of $ABCD$ is 6×3=18 cm^2.
Hence the sum of shaded areas is 18－(1+9+4)=4 cm^2.

Answer：(B)

5. A triangle is formed with 10 matchsticks of equal length connected end to end. No matchsticks are bent or broken. How many different triangles can be formed?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

【Suggested solution】

By the Triangle Inequality, the sum of two sides of a triangle is greater than the third side. The only possible triangles have side lengths (2, 4, 4) and (3, 3, 4).

Answer：(A)

6. A piece of paper in the shape of parallelogram is folded into two with the crease bisecting the area of parallelogram. How many different kinds of origami methods are possible?

(A) 0 (B) 1 (C) 2
(D) 3 (E) infinitely many

【Suggested solution】

By symmetry of parallelogram, the crease will pass through the center of parallelogram and it bisects the area of parallelogram. Therefore there is infinitely many kinds of origami methods, so E.

Answer：(E)

7. A TV company plans to broadcast a series with 48 episodes. One episode is aired each day except on Saturday and Sunday. If the first episode is aired on Thursday, on what day of the week will the last episode be aired?

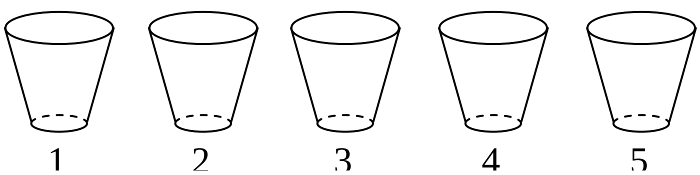
(A) Monday (B) Tuesday (C) Wednesday (D) Thursday (E) Friday

【Suggested solution】

We can deduce the second episode is aired on Friday, the third on Monday, the forth episode on Tuesday, the fifth episode on Wednesday, the sixth episode on Thursday; and 5 episodes forms a cycle. Since 48=5×9+3, so episode 48 (the last episode) will be aired on Monday.

Answer：(A)

8. Cups labelled 1, 2, 3, 4 and 5 with mouth upwards line in row, as shown below. Initially a ball is put into cup #3. In each move, the ball is transferred to an adjacent cup. If the ball is in cup #1, it can only be moved to cup #2. If the ball is in cup #5, it can only be moved to cup #4. After $2^{10}+3^{8}$ moves, which of the following statements about the ball is correct?



(A) It cannot be in cup #3, cannot be in cup #4 and cannot be in cup #5
(B) It cannot be in cup #2, cannot be in cup #4 and cannot be in cup #5

（C） It cannot be in cup #1, cannot be in cup #4 and cannot be in cup #5

（D） It cannot be in cup #1, cannot be in cup #3 and cannot be in cup #5

（E） It cannot be in cup #2 and cannot be in cup #4

【Suggested solution】

Since the ball is transferred to an adjacent cup in each move, the parity of the cup label changes. After $2^{10}+3^{8}$ moves, the parity has changed. So the ball will be in a cup with an even label. Thus the ball is not possible to be in cup #1, cup #3 and cup #5.

Answer：（D）

9. During the holidays, Dick worked part-time washing bowls in a restaurant. He got paid 3 dollars for washing one bowl. If he broke a bowl, he got no pay for washing it, and must pay 9 dollars to the owner. In one week, Dick washed 500 bowls and earned 1368 dollars. How many bowls did he break?

（A） 7 （B） 8 （C） 9 （D） 10 （E） 11

【Suggested solution 1】

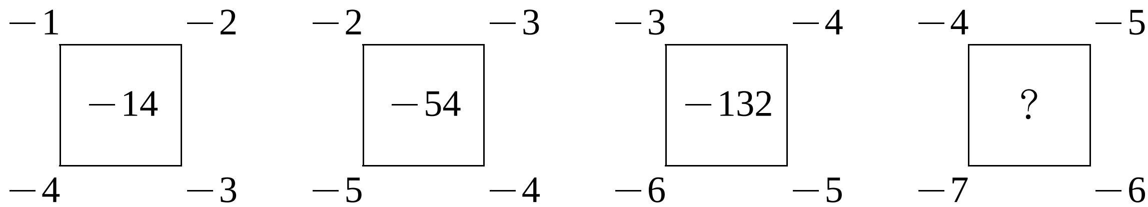
Let Dick has broken x bowls，which means he finished washing $500-x$ bowls，from the meaning of question , we get $3\times(500-x)-9x=1368$，then $x=11$.

【Suggested solution 2】

After washing 500 bowls, Dick should receive 3×500=1500 dollars, but he only got 1368 dollars, which was 132 dollars less. For each bowl Dick broke, he lost 3 dollars of income and 9 dollars in compensation, for a total of 12 dollars. Thus Dick had broken 132÷12=11 bowls.

Answer：（E）

10. The diagram below shows four squares with numbers which exhibit a certain pattern. What number should be inside the fourth box?



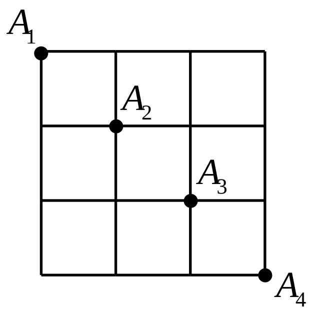
（A） −210 （B） −260 （C） −288 （D） −308 （E） −330

【Suggested solution】

By observation, the number inside the box is equal to a product of the sum of the two lower numbers times the product of the two upper numbers. Hence the number need to be filled inside the fourth box is $(-4)\times(-5)\times((-6)+(-7))=-260$.

Answer ：（B）

11. The diagram below shows a square network of roads, A_1, A_2, A_3 and A_4 are four intersections on the same diagonal. We want to go from A_1 to A_4 by going only to the east or to the south, without passing through A_3. How many different paths are there?

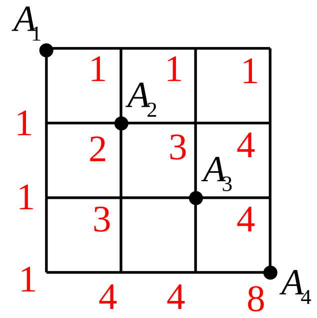


（A） 8 （B） 10 （C） 20

（D） 15 （E） 12

【Suggested solution 1】

When a person walks from point A_1 to A_4, he needs to walk three times each to the right and downwards. If not considered the constraint that cannot pass through point A_3, number of ways should be $C_3^6=20$. Since there are $C_2^4 \times C_1^2=12$ ways to get from point A_1 to A_4 passing through point A_3, number of ways for a person to walk from point A_1 to A_4 not passing through point A_3 is 20－12=8.



【Suggested solution 2】

The number next to each point represents number of ways to get there from point A_1 as shown in the figure. Thus there are 8 ways for a person to walk from point A_1 to A_4 not passing through point A_3.

Answer：(A)

12. Each row in a cinema has 80 seats, and row 13 to row 24 are reserved for students from a secondary school. There are 15 empty seats in these rows when all the students have taken their seats. How many secondary school students went to the cinema?

(A) 945 (B) 875 (C) 865 (D) 775 (E) 765

【Suggested solution】

There are 24－13+1=12 rows from row 13 to row 24, and each row has 80 seats. The total number of seats is 80×12=960. Since there are 15 empty seats, the total number of secondary school students is 960－15= 945.

Answer：(A)

13. The total weight of 3 apples is equal to that of 4 bananas, and the total weight of 5 bananas is equal to that of 6 oranges. How many apples have the same total weight as 16 oranges?

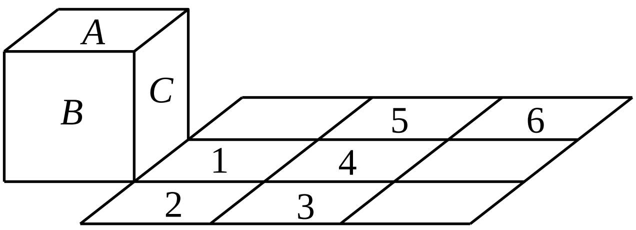
(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

【Suggested solution】

From the question, we can deduce that 20 bananas have the same weight as 15 apples. Thus 20 bananas will have the same weight as 24 oranges. Hence, 15apples will have the same weight as 24 oranges. Thus 10 apples will have the same weight as 16 oranges.

Answer：(E)

14. The diagram below shows a cube with three of its faces labelled *A*, *B* and *C*, and a 3×3 square with six of its squares labelled 1, 2, 3, 4, 5 and 6. The cube is tipped over so that face C lies on square 1, tipped over again so that face B lies on square 2, and so on until the cube lies on square 6. What is the sum of the numbers of the squares on which the cube has laid with face *B* on top?



(A) 2 (B) 6 (C) 7 (D) 9 (E) 10

【Suggested solution】

When the cube is tipped onto square 1, face B is in front. When the cube is tipped onto square 2, face B is at the bottom. When the cube is tipped onto square 3, face B is on the left. When the cube is tipped onto squares 4 and 5, face B remains on the left. When the cube is tipped onto square 6, face B is on top.
Hence the desired sum is 6.

Answer：(B)

15. A deck of 54 cards has 2 jokers, and 13 cards of each of spades, hearts, clubs and diamonds. At least how many cards should be drawn at random so that there are at least 4 cards of the same suit?

(A) 54 (B) 14 (C) 15 (D) 16 (E) 17

【Suggested solution】

Consider the worst situation that we have 2 jokers and 3 cards from each suit to be drwan. There are in total 14 cards but still did not give us a suit with more than 4 cards. And when one more card is drawn, we will have 4 cards for a certain suit, by piageon hole principle. Thus the answer is 15 cards.

Answer：(C)

16. The number 5^n+7^n is divided by 100 where n is any non-negative integer. How many different values of the remainder are there?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 15

【Suggested solution】

When $n = 0$, the last two digits of 5^n+7^n is 02. When n=1, the last two digit of 5^n+7^n is 12. When $n\geq 2$, the last two digits of 5^n is 25, whereas the last two digits of 7^n can only be 01, 07, 49 or 43. The sum of the last two digits can only be 26 or 32 or74 or 68. Hence there are 6 possibilities.

Answer：(B)

17. For any positive integers a and b, define a new operation $a\odot b$ which yields the remainder when the larger of a and b is divided by the smaller one. For example, $5\odot 12 = 12\odot 5 = 2$. Given that $(19\odot x)\odot 19 = 5$, what value below is not possible for x?

(A) 12 (B) 26 (C) 33 (D) 39 (E) 45

【Suggested solution】

If x=12, then $(19\odot 12)\odot 19=7\odot 19=5$; If x=26, then $(19\odot 26)\odot 19=7\odot 19=5$;
If x=33, then $(19\odot 33)\odot 19=14\odot 19=5$; If x=39, then $(19\odot 39)\odot 19=1\odot 19=0$;
If x=45, then $(19\odot 45)\odot 19=7\odot 19=5$.
Only the number 39 does not satisfy the condition.

Answer：(D)

18. What is the total number of positive integers consisting of three different digits in which the tens digit is equal to the units digit of the sum of the other two digits?

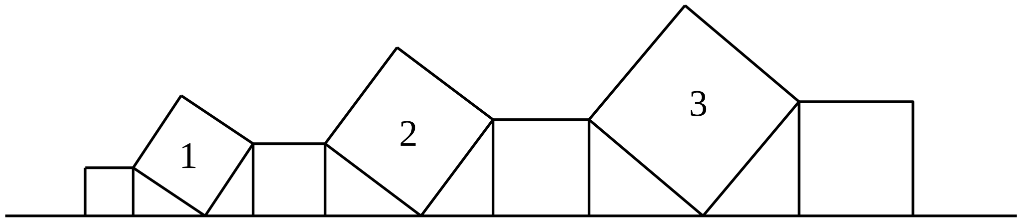
(A) 36 (B) 60 (C) 72 (D) 90 (E) 108

【Suggested solution】

From the given condition, we know that the hundreds digit and units digit are different and cannot be zero. There are no other constraints. Thus the total number is 9×8=72.

Answer：(C)

19. The diagram below shows seven squares resting on a straight line. The areas of three tilted squares are 1, 2 and 3. What is the total area of the other four squares?



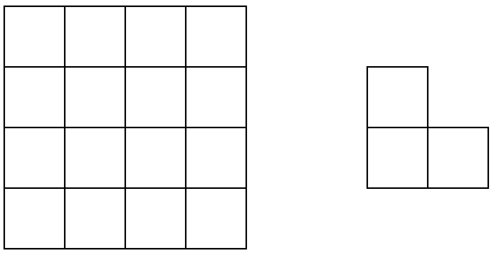
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

【Suggested solution】

From Pythagorean Theorem and properties of congruent triangles, it is deduce that the area of each tilted square equal to the sum of its adjacent squares. So the total area of the other four squares is 4.

Answer：(A)

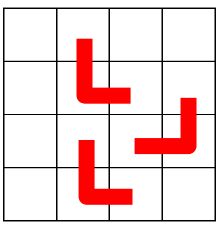
20. On a 4×4 chessboard shown in the diagram below on the left, we wish to place a minimum number of copies of the shape shown in the diagram below on the right, so that no more copies of this shape can be placed. Copies may be rotated. What is this minimum number of copies?



(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

【Suggested solution】

Divide the 4×4 chessboard into four 2×2 chessboards. The copies of the shapes placed must cover at least 2 squares of each 2×2 chessboard in order to prevent another copy of the shape to be placed within the 2×2 chessboard. Thus we must cover at least 8 squares, which requires at least 3 copies. These may be placed as shown in the diagram below.



Answer：(B)

21. We place 100 table tennis balls inside n boxes so that the number of balls in each box contains the digit 8, such as 8 balls, 18 balls, 83 balls and 88 balls. When n=3, the number of table tennis balls in the boxes are 8, 8 and 84 respectively. If n = 5, and two of the boxes have the same number of balls while other boxes have different number of balls, what is the largest total number of balls in two boxes?

【Suggested solution】

First, the least number of balls in each box is 8, so no box will have number of balls more than 100－8×4=68. So the number of balls will have "8" in the unit digit.

And since there are two boxes having same number of balls, the total number of balls is at least 8+8+ 18+28+38=100. And this is the only case. The sum of number of balls in the two boxes with most of the balls is 28+38=66.

Answer：066

22. Let a, b, c and d be positive integers less than 10, and x be an integer such that $ax^3-bx^2-cx-d=0$. What is maximum value of x?

【Suggested solution】

If $x\geq 10$ ，then

$$x^3\geq 10x^2\geq (b+1)x^2=bx^2+x^2\geq bx^2+10x$$

$$\geq bx^2+(c+1)x\geq bx^2+cx+10>bx^2+cx+d$$

So x must be less than 10. Also, when a=1, b=c=8, d=9, x = 9, the requirement are satisfied. Thus the maximum value of x is 9.

Answer：009

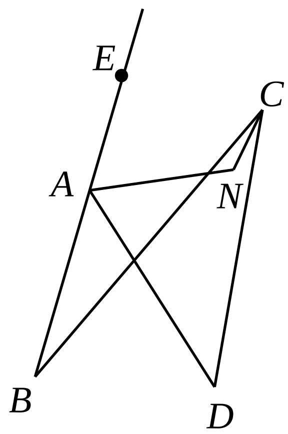
23. Let a, b and c be real numbers such that $a+b+c=0$ and $abc=-15$. What is the value of $a^2(b+c)+b^2(c+a)+c^2(a+b)$?

【Suggested solution】

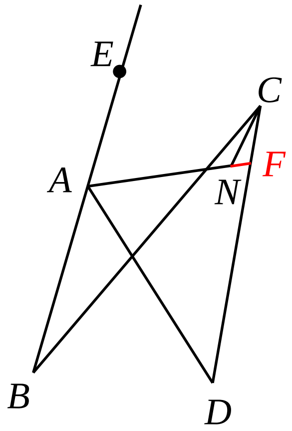
$$\begin{aligned}a^2(b+c)+b^2(c+a)+c^2(a+b)&=a^2b+a^2c+b^2c+b^2a+c^2a+c^2b\\&=ab(a+b)+bc(b+c)+ca(a+c)\\&=ab(-c)+bc(-a)+ca(-b)\\&=-3abc\\&=45\end{aligned}$$

Answer：045

24. The diagram below shows four lines segments AB, BC, CD and DA on the plane where $\angle ABC=24°$ and $\angle ADC=42°$. Point E is on the extension of line BA, and the angle bisectors of $\angle DAE$ and $\angle BCD$ intersects at point N. What is the measure, in degrees, of $\angle ANC$?



【Suggested solution】



As in the diagram, *AD* and *BC* intersects at point *O*, by exterior angle of triangle,

$$\angle BOD = \angle BCD + \angle CDO = \angle BCD + 42°$$

Then $\angle BAD = \angle BOD - \angle ABO = \angle BCD + 18°$

Thus $\angle EAD = 180° - \angle BAD = 162° - \angle BCD$

As N is the intersection point of two angle bisectors，

$$\angle DAN = \frac{1}{2}\angle EAD = 81° - \frac{1}{2}\angle BCD \text{ 、 } \angle NCD = \frac{1}{2}\angle BCD$$

Extend *AN* to meet *CD at the* point *F*,

$$\angle ANC = \angle NCF + \angle NFC = \angle NCD + \angle DAN + \angle ADC = \frac{1}{2}\angle BCD + 81° - \frac{1}{2}\angle BCD + 42°$$

$$= 123°$$

Answer：123

25. In the expression 6□7□8□9, an arithmetic sign (plus, minus, multiplication or division sign, can be used with repetition) is placed in each bracket □. Open bracket is allowed (it is optional). What will be the largest 3- digit number obtained?

【Suggested solution】

It is not hard to see that subtraction and division signs should not be used. Now a multiplication sign must be filled in the last □; otherwise the maximum value will not exceed 6 × 7 × (8 + 9) = 714. If the two □s in the front are filled with multiplication signs, then the number obtained is 3024, which is not a three-digit number. If both are filled with addition signs, then the largest value which may be obtained is (6 + 7 + 8) × 9 = 189. Suppose one is filled with an addition sign and the other a multiplication sign. For 6 + 7 × 8 × 9, the maximum value which may be obtained is (6 + 7) × 8 × 9 = 936. For 6 × 7 + 8 × 9, the maximum value which may be obtained is 6 × (7 + 8) × 9 = 810. Hence the largest 3 digit number is 936.

Answer：936