2011 JUNIOR DIVISION FIRST ROUND SOLUTION

1. $2011+1102\times(1-3)=2011+1102\times(-2)=2011-2204=-193$ or $2011+1102\times(1-3)=2011+1102-1102\times3=3113-3306=-193$.

   Answer：(D)

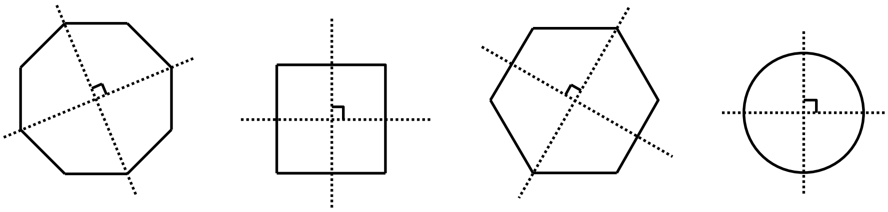
2. Since $\pi=3.14159\cdots$, $\dfrac{22}{7}=3.14285\cdots$, $304\%=3.04$, the relationship of these five numbers are $\dfrac{22}{7}>\pi>3.14>3.135>304\%$. Obviously $\dfrac{22}{7}$ is the largest.

   Answer：(C)

3. From $-223-(-253)=-223+253=30$, the temperature facing the sun is higher than behind the sun by 30°C.

   Answer：(A)

4. By unfolding ①, two perpendicular lines are shown. Obviously these two lines are axis of symmetry of figure①, thus figure① have centre of symmetry.



Octagon, quadrilateral, hexagon and circle have the possibility of being symmetry figures with centre of symmetry found. Triangles are not symmetry with the centre as symmetry.

   Answer：(D)

5. If BC is represented by negative numbers, then $2011-(-550)=2561$.Since there is no year 0, so $2561-1=2560$ years.

   Answer：(B)

6. Using any line parallel to the side of a rectangle, one can get different cylinders. As there are infinitely many lines that are parallel to the length or width of the rectangle, there are infinitely many possible cylinders generated.

   Answer：(E)

7. Through observation, the following pattern is found: from the first graph, the small square inside the large square appeared anti-clockwisely in four different corners of the large square. The pattern has a period of 4. Since $2011=4\times502+3$, the 2011th figure is the same as the third one.

   Answer：(A)

8. 【Solution1】

   $\angle AOC=\angle AOD-\angle DOC=123°-90°=33°$,

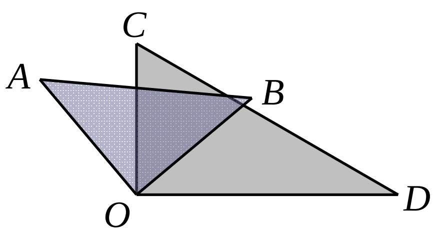
   then $\angle BOC=\angle AOB-\angle AOC$,

   so $\angle BOC=90°-33°=57°$.

   【Solution 2】

   $\angle BOD=\angle AOD-\angle AOB=123°-90°=33°$,

   then $\angle BOC=\angle DOC-\angle BOD$, so $\angle BOC=90°-33°=57°$.



   Answer：(C)

9. 【Solution1】

The apples brought by Leith exceed 3kg. For the first 3 kg, each kg costs$6. When exceeds 3 kg, each kg costs $6\times(1-20\%)$ = \$4.8. Therefore, Leith should pay $3\times6+(8-3)\times6\times(1-20\%)=18+5\times6\times0.8=\$42$.

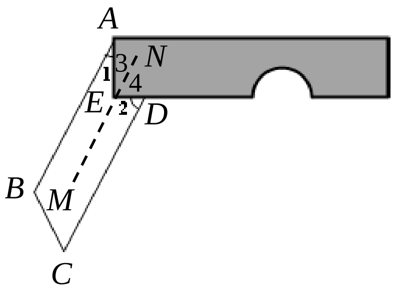
【Solution 2】

The apples brought by Leith exceed 3kg. The part that exceeds 3kg has a discount of 20%, so he should pay money for $(8-3)\times(1-20\%)=4$ kg apples instead of his additional 5 kg. Overall, he should pay for 3+4=7 kg apples, thus the price is $6\times7=\$42$.

Answer：(C)

10. As shown on the right figure, MN be a line passes through E and $MN//AB$, then $\angle1=\angle3$. And $AB//CD$, so $MN//CD$, we can get $\angle2=\angle4$. Therefore, $\angle1+\angle2=\angle3+\angle4=\angle AED=90°$.

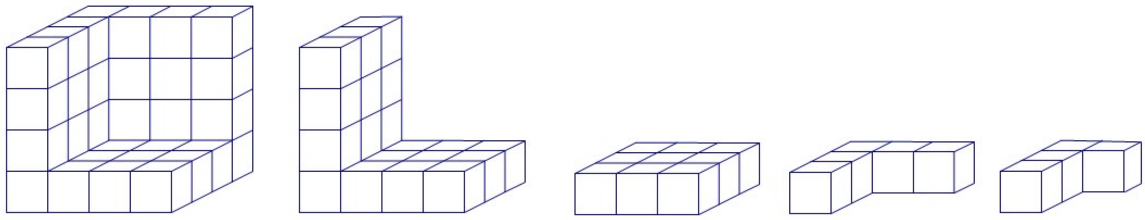
Answer：(D)



11. The company expects to sell 5400+6000+6600=18000 pieces in the last quarter sale. However, the actual sale is $5400\times84\%+6000\times87\%+6600\times93\%=15894$ pieces. Thus the achievement percentage equals to $\frac{15894}{18000}\times100\%=88.3\%$.

Answer：(B)

12. By thinking backward, consider what will remain after cutting the 5 blocks from the 4×4×4 block. After the second block is cut, what remains is combination of three 1×4×4 blocks. (Some overlapping may happen, refer to figure1). After the first block is cut, what remains is combination of two1×3×4 blocks (there is an overlapping of 1×1×3, refer to figure2). And after cutting the third block, what remains is a 1×3×3 block (see figure 3). Then, the fourth block, what remains is a block with L shaped (figure 4). Finally, after cutting the fifth block, we need to cut a 1×1×1block from the L shaped to ensure the remaining cubes forms one block. What is left is a block shown in figure 5.



(1) (2) (3) (4) (5)

Answer：(C)

13. In these six pieces, the area of the two large triangles occupies half of the space of that square. As shown from the graph, only these two triangles are not shaded, the area of the shaded region should equal to half of the area of that square, so $\frac{1}{2}\times 40^2 = 800$ cm².

Answer：(D)

14. Let the date be x and $24 > x > 7$. In the same vertical column, the upper and the lower date can be written as ($x-7$) and ($x+7$), thus $x-7+x+x-7=3x>21$. Which means the sum of that three dates should be a multiple of 3 and greater than 21. Hence we circle 11, 18 and 25 to get 54.

Answer：(E)

15. 【Solution 1】

Let the length of small cube be a cm, then $6a^2\times 8-(2a)^2\times 6=216$, $24a^2=216$, we can get a=3 cm.

【Solution 2】

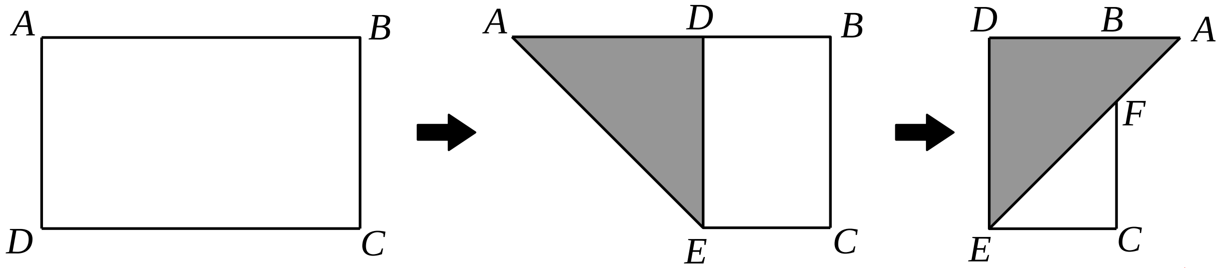
Since only three faces of the small cube are exposed, the other three faces are embedded inside. For eight small cubes, total number of embedded faces are $8\times 3=24$, which equals to the loss of surface area. Let the length of small cube be a cm, then $24a^2=216$, so a=3 cm.

Answer：(B)

16. According to instruction, that player score 39 points by field goal (2-point shot) and three-pointer (3-point shot). As 39 is an odd number, the number of three-pointer should be an odd number. Thus, he may get 1 three-pointer and 18 field goals; or 3 three-pointers and 15 field goals; or 5 three-pointers and 12 field goals; or 7 three-pointers and 9 field goals. (If the number of getting three-pointer equals to nine, then the number of getting field goal will be less than three-pointer, which does not satisfy the condition). Hence cases in getting field goal and three-pointer are 1+18=19, 3+15=18, 5+12=17 or 7+9=16. It follows that the total times will not equal to 15.

Answer：(A)

17. Since AD=6 cm, AB=10 cm, thus BD=4 cm. In the rightmost figure, $BC//DE$, we get $\frac{AB}{AD}=\frac{BF}{DE}$, and $AB=6-4=2$ cm. Hence BF=2 cm. It follows that $S_{\Delta ABF}=\frac{1}{2}AB\times BF=2$ cm².



Answer：(A)

18. The route of the remote-controlled car forms a regular polygon and each of its exterior angle equals to 30°. According to the sum of exterior angle of the polygon, we know that the number of sides is 12, thus the remote-controlled car travels 12 m.

Answer：(C)

19. Assume n to be the side of this convex polygon. As each interior angle is greater than 100° and smaller than140°, $100n<(n-2)180<140n$. Hence $4.5<n<9$. So the number of sides of this polygon could not be 9.

Answer：(E)

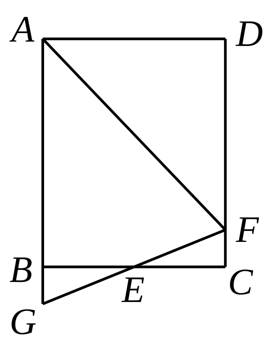
20. Since $3+7+15+31+63=119<180$, the number that multiply 127 should be 1. Also, as $63+127>180$, the number that multiply 63 should be 0. Hence, the number that multiplies 63 is on the vertex *R* or *T*. This can induce that he should start from *P* or *T*. If he starts from *P*, the number obtained is 168, not180. If he starts from *T*, the number obtained is
$0\times3+1\times7+1\times15+1\times31+0\times63+1\times127=180$. As a result, he starts from *T*.

Answer：(D)

21. From the conditions, we know that the drunkard walks east/west in his 1, 3, 5 and 7 trials. While he move north/south in his 2, 4 and 6 trials. Therefore at most he can walk 1+3+5+7=16 m in the east/west direction as well as 2+4+6=12 m in the north/south direction. By Pythagorean Theorem, his furthest distance from his starting point is $\sqrt{16^2+12^2}=20$ m.

Answer：020

22. Let the length of segment *CF* be *x*. As *BG*//*CF*, and *E* is the mid point of *BC*, it is easy to show that $\triangle BEG\cong\triangle CEF$，thus *BG*=*CF*=*x*.
Since $\angle AFE=\angle CFE=\angle AGE$, hence *AG*=*AF*.
Also, as *AG*=*AB*+*BG*=25+*x*, thus *AF*=25+*x*. In addition, by $AD=20$, $DF=25-x$, the right triangle *ADF* and the Pythagorean Theorem, we get $(25+x)^2=(25-x)^2+20^2$，then 100*x*=400, so *x*=4.



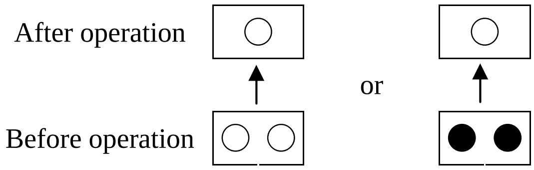
Answer：004

23. There are 120 ($5\times4\times3\times2\times1=120$) cases for the integral part to be one-digit. Then we consider those with 2-digit in the integral part. Once we fix the integral part, the number of permutations of 3 digits in the decimal place equals $3\times2\times1=6$.The two-digit numbers of the integral part arranged in ascending order are 12, 13, 14, 15, 21, 23, …Thus the 145th number is 21.345 and the150th number is 21.543, their difference is 0.198, which is 198 when multiplied by 1000.

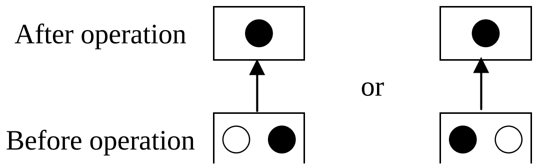
Answer：198

24. By working backward, we can consider the status of the counters before each operation.

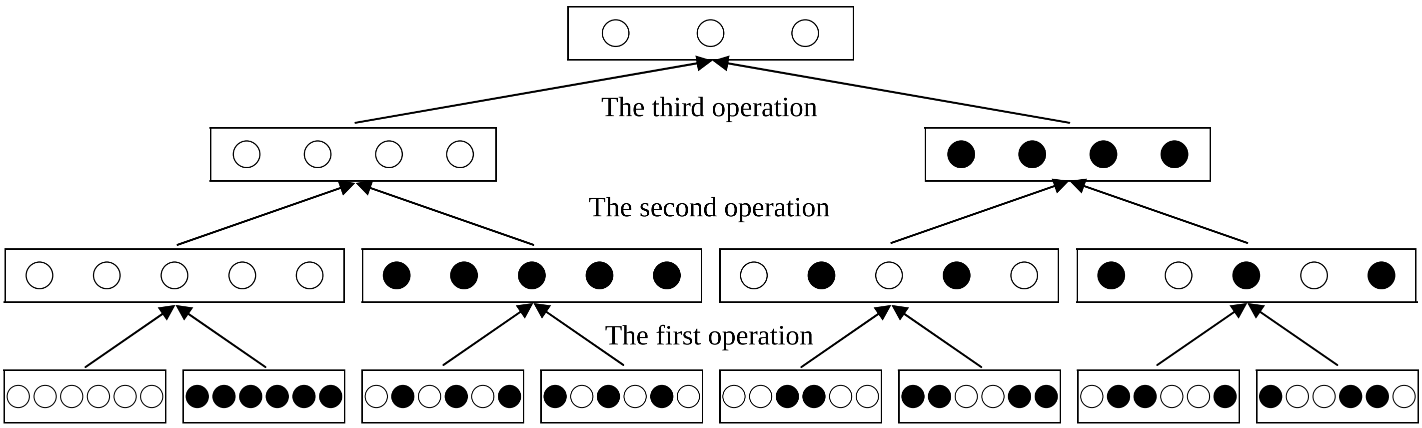
If there is one white counter after an operation, there should be two counters with same colour before the operation.



If there is one black counter after an operation, there should be two counters with different colours before the operation.



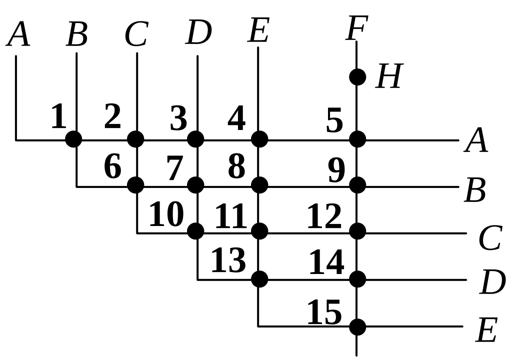
Using the above process, we can deduce the following：



At the beginning of the game, there are 8 different combinations of 6 counters.

Answer：008

25. Note that every line has 5 interchange stations. When traveling on one line, at most two changes are made. Thus, Mickey needs to ride at least 3 times on each line. For the line that come-across his home, he needs to ride four times. (Since it starts and ends in the station near his home, and there will not be change). As a result, Mickey needs to ride at least $3\times5+4=19$ times. In other words, he needs to make 18 changes. Refer to the figure on the right, it shows six lines *A*, *B*, *C*, *D*, *E*, *F* and interchange stations 1 to 15. Mickey starts at H, he can take the following route, which included 18 changes to go back to station H.



H→15→4→1→9→14→3→2→12→5→4

→13→10→6→8→11→10→7→9→H

Answer：018