International Mathematics Assessments for Schools

2019 ~ 2020 JUNIOR DIVISION FIRST ROUND PAPER

Time allowed：75 minutes

When your teacher gives the signal, begin working on the problems.

INSTRUCTION AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. No calculators, slide rules, log tables, math stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 20 multiple-choice questions, each with 5 choices. Choose the most reasonable answer. The last 5 questions require whole number answers between 000 and 999 inclusive. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a mathematics assessment, not a test; do not expect to answer all questions.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.

THE ANSWER SHEET

1. Use only pencils.
2. Record your answers on the reverse side of the Answer Sheet (not on the question paper) by FULLY filling in the circles which correspond to your choices.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places. So please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The IMAS reserves the right to re-examine students before deciding whether to grant official status to their scores.

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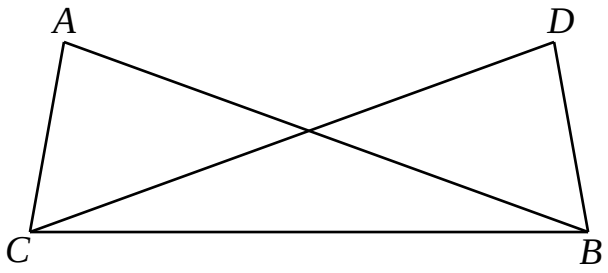
Questions 1-10, 3 marks each

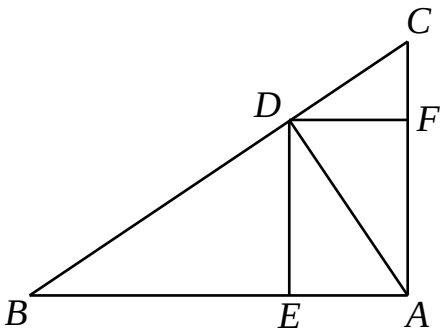
1. What is the simplified value of $(2019-2020)^{2020}+(2018-2019)^{2019}$?
   （A） 0　（B）−2019　（C）−1　（D）−2020　（E）2

2. The figure below shows $AB=BC=CD$, $AC=BD$ and $\angle A=80°$. What is the size of $\angle ACD$, in degrees?



   （A）20　（B）30　（C）45　（D）50　（E）60

3. In triangle *ABC*, $\angle A$ is the right angle. The line through *A* perpendicular to *BC* intersects *BC* at *D*. Lines through *D* intersect *AB* and *AC* perpendicularly at *E* and *F*, respectively. What is the total number of right triangles in the figure?



   （A）5　（B）6　（C）7　（D）8　（E）9

4. One third of an acute angle and its complementary angle sums up to $60°$, what is the supplementary angle of this acute angle, in degrees?
   （A）45　（B）60　（C）90　（D）120　（E）135

5. There is an integer *n*, such that $2020\times(\frac{10}{101})^n$ is an integer. How many possible *n's* are there?
   （A）1　（B）2　（C）3　（D）4　（E）5

6. The school restaurant sells three kinds of lunch boxes, the prices of which are \$15, \$13 and \$11, respectively. On a certain day, the total income of the restaurant is \$2019. Which of the following could be the possible number of lunch boxes sold on that day?

（A）105　（B）130　（C）155　（D）185　（E）205

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7. In an exam, 25 students obtained excellent scores in Language. The number of students with excellent scores in math is 2 more than that in the Language. The students with both excellent scores in Math and Language scores are one third of all students. Every student obtained at least one excellent score. How many students are there in total?

（A）78　（B）52　（C）42　（D）40　（E）39

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8. Define the operation $a*b=\dfrac{a+b+2019}{a-b+2019}$. If $a*2019=7$, what is the value of a?

（A）673　（B）674　（C）1009.5　（D）1346　（E）2019

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9. Non-negative integer x satisfies $|2x+5|\le 15\times(1+x+x^2+...+x^{2019})^0$. What is the sum of all such x's?

（A）15　（B）25　（C）35　（D）45　（E）55

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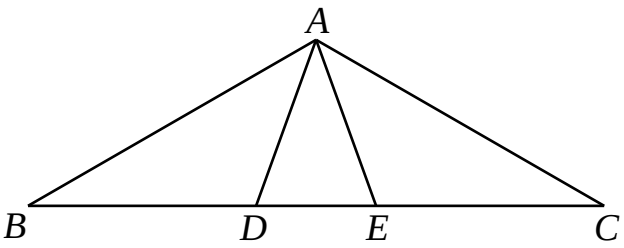
10. In a triangle ABC, $AB=AC$. Line segments AD and AE trisect $\angle BAC$, as shown in the figure. If $\angle ADB=110°$, what is $\angle ABC$ in degree?



（A）30　（B）35　（C）40　（D）45　（E）60

Questions 11-20, 4 marks each

11. When $x=5$, mx^5+nx^3+kx+2 is 100. What is the value of mx^5+nx^3+kx+2 when $x=-5$?

（A）−100　（B）−99　（C）−98　（D）−97　（E）−96

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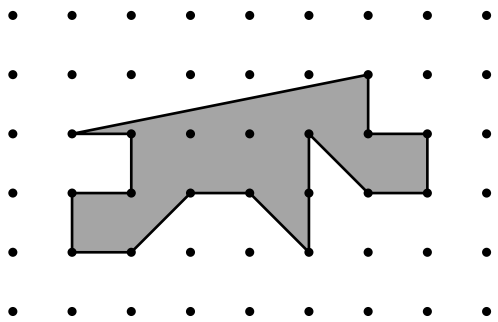
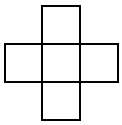
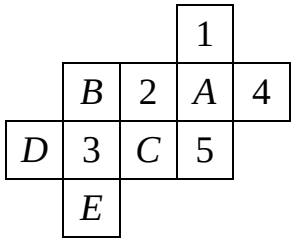
12. In the figure below, the distance between two adjacent vertices is 1 cm. What is the area of the shaded portion of the figure, in cm^2?



（A）7.5　　（B）8　　（C）8.5　　（D）9　　（E）9.5

13. Each square in the figure below is filled with a positive integer so that the sum of each  (five squares) is divisible by 3.



What is the minimum value of $A+B+C+D+E$?

（A）3　　（B）6　　（C）8　　（D）9　　（E）15

14. Ten-digit number $\overline{A20192020B}$ is divisible by 9, where A, B are digits. If the quotient has the ten-thousands digit 0, what is the value of $|A-B|$?

（A）0　　（B）1　　（C）2　　（D）3　　（E）4

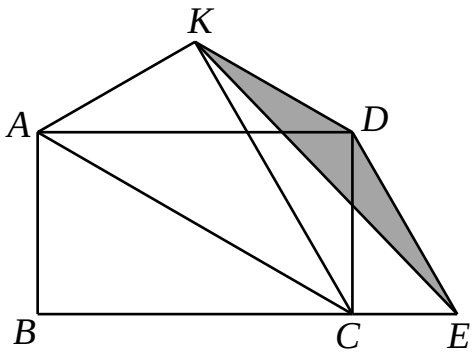
15. Let x be the integers such that $\dfrac{21}{2+x}$ is an integer. How many such x's are there?

（A）2　　（B）3　　（C）4　　（D）6　　（E）8

16. Given that $|a|=3$, $|b|=5$, $|c|=7$, $|d|=11$, what is the least possible value of the expression $ab+ac+ad+bc+bd+cd$?

（A）0　　（B）–52　　（C）–94　　（D）–100　　（E）–102

17. In rectangle *ABCD*, $AB = \sqrt{3}$ cm, $BC = 3$ cm. Put a right triangle *DCE* outside *ABCD* and along edge *CD* and $\angle CDE = 30°$. Flip along *AC* such that *B* lands at *K*. Join *KA*, *KC*, *KE*, *KD*, as in the figure. What is the area of the shaded region, in cm^2?



(A) $\frac{3}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{3}$ (D) $\sqrt{3}-1$ (E) $\frac{2\sqrt{3}}{3}$

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18. Given that $x = \sqrt{19-8\sqrt{3}}$, which of the following equalities holds true?

(A) $x^2 = 11$ (B) $x^2 + 8x = 13$ (C) $x^2 - 8x = -13$
(D) $x^2 + 4x = 8$ (E) $x^2 - 4x = -8$

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19. Mickey has a large gridded paper. Each small square has side of length 1 cm. He draws a circle with a center at some vertex of a square with radius 6 cm. How many small whole squares are inside the circle?

(A) 64 (B) 88 (C) 96 (D) 100 (E) 144

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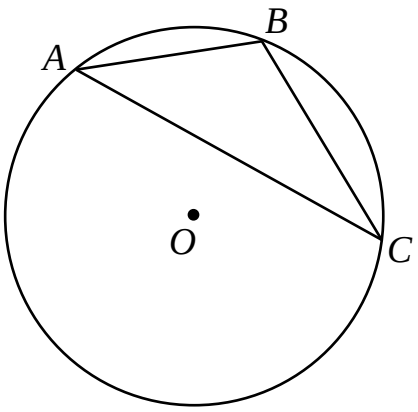
20. Suppose $a, b, c, d, e, f, g, h, i, j$ is an arrangement of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 with different letters representing different numbers. At most how many of the ten fractions $\frac{a}{b}, \frac{b}{c}, \frac{c}{d}, \frac{d}{e}, \frac{e}{f}, \frac{f}{g}, \frac{g}{h}, \frac{h}{i}, \frac{i}{j}, \frac{j}{a}$ can be reduced to integer values?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Questions 21-25, 6 marks each

21. Given *A*, *B*, *C* on circle *O* as shown in the figure, chord $AB = 10$ cm and $\angle ACB = 30°$. What is the area in cm^2 of circle *O*? (Assume $\pi = 3.14$)



22. Four prime numbers (may not be distinct) *A*, *B*, *C*, *D* satisfy that $(A+1)(A^2+B^2+C^2+D^2) = 1149$. What is the value of $A \times B \times C \times D$?

23. Two distinct numbers are selected from 1, 2, 3, …, 100 such that their sum is a factor of 2020. How many different methods of selections are there in total?

24. Integer *a* and real number *x* satisfy $x^2 - ax + 1 = 0$. What is the minimum value of $\left| x^4 + \frac{1}{x^4} - 2019 \right|$?

25. Arrange the ten numbers 2011, 2012, 2013, …, 2020 randomly on the circumference of a circle, and compute the greatest common divisor of every two adjacent numbers, then add these ten greatest common divisors. What is the maximum of this sum?

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