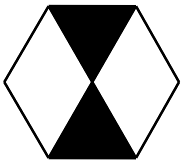
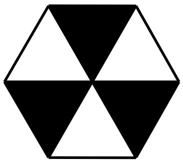
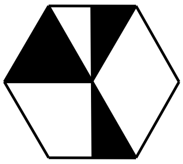
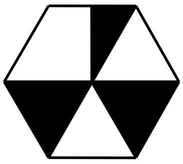
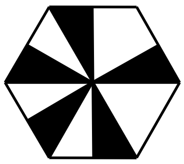
Solution Key to Second Round of IMAS 2016/2017
Middle Primary Division

1. Among the numbers 5, 7, 11, 15 and 19, which number cannot be expressed as a sum of two prime numbers?
   （A）5 （B）7 （C）11 （D）15 （E）19

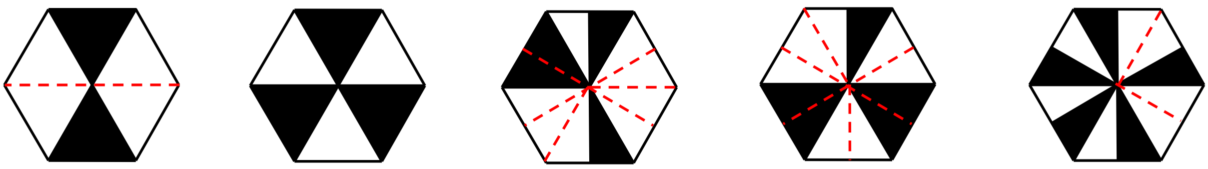
【Suggested Solution】
Note that $5=2+3$, $7=2+5$, $15=2+13$ and $19=2+17$.
Suppose that 11 can be expressed as a sum of two prime numbers, then one must be an odd and the other must be an even. Since the only even prime number is 2, the other number is 9, which is not a prime number, thus, a contradiction. Therefore the answer is（C）

Answer：（C）

2. There is a dart game in an amusement park where the goal is to hit the black portions of the regular hexagon in order to win a prize. From the choices below, which dart board will give the highest chance of winning?

（A）

（B）

（C）

（D）

（E）


【Suggested Solution】
By drawing line segments on figures (A), (B), (C), (D) and (E): (refer to the figures below.).


(A) (B) (C) (D) (E)

Observe that the shaded area of figure (A) is $\frac{2}{6}=\frac{4}{12}$, shaded area of (B) is $\frac{3}{6}=\frac{6}{12}$, shaded area of (C) is $\frac{4}{12}$, shaded area of (D) is $\frac{5}{12}$, and shaded area of (E) is $\frac{5}{12}$.

Since $\frac{6}{12}>\frac{5}{12}>\frac{4}{12}$, therefore, (B) has the best chance to win a prize.

Answer：（B）

3. What is the units digit of the value of $\underbrace{2\times 2\times\cdots\times 2}_{2016\text{ terms}}+\underbrace{3\times 3\times\cdots\times 3}_{2017\text{ terms}}$?

(A) 1 (B) 3 (C) 6 (D) 7 (E) 9

【Suggested Solution】

The units digit of $\underbrace{2\times 2\times\cdots\times 2}_{n\text{ terms}}$ cycles through the numbers 2, 4, 8 and 6. While the units digit of $\underbrace{3\times 3\times\cdots\times 3}_{n\text{ terms}}$ cycles through 3, 9, 7 and 1. Since $2016=4\times 504$, then the units digit of $\underbrace{2\times 2\times\cdots\times 2}_{2016\text{ terms}}$ is 6. On the other hand, $2017=4\times 504+1$, then the units digit of $\underbrace{3\times 3\times\cdots\times 3}_{2017\text{ terms}}$ is 3. Therefore the units digit of the value of the given expression is $6+3=9$.

Answer：(E)

4. Alex bought a total of nine pencils and ball pens. One ball pen costs \$3 each, while one pencil costs \$2 each. If Alex spent a total of \$22, how many ball pens did he buy?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

【Suggested Solution 1】

If Alex bought 9 ball pens, he should have spent a total of $9\times\$3=\$27$, which is \$5 more than his actual spent. Since the difference of the price of a ball pen and the pencil is $\$3-\$2=\$1$, then the number of pencils bought is $5\div 1=5$. Therefore the number of ball pens bought is $9-5=4$.

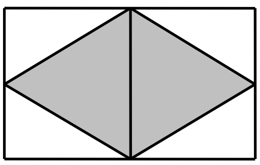
【Suggested Solution 2】

If Alex bought 9 pencils, he should have spent a total of $9\times\$2=\$18$, which is \$4 less than his actual spent. Since the difference of the price of a ball pen and the pencil is $\$3-\$2=\$1$, then the number of ball pens bought is $4\div 1=4$.

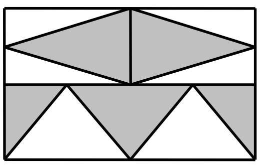
Answer：(D)

5. In each of the following options, the dimensions of the rectangle are 10 cm by 6 cm. There are some shaded triangles inside each rectangle. The vertices of the shaded triangles must be either at the endpoints of a line segment or points that divide it into equal parts. From the options below, which figure has the largest shaded region?

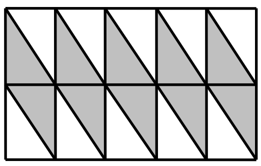
(A)



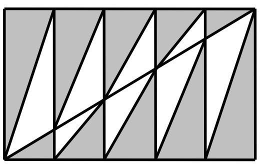
(B)



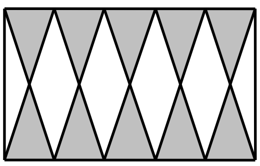
(C)



(D)



(E)



【Suggested Solution 1】

By finding the areas of the shaded triangles.

Shaded area of figure (A) is $2\times\frac{1}{2}\times(10\div2)\times6=30\,\text{cm}^2$；

Shaded area of figure (B) is

$2\times\frac{1}{2}\times(10\div2)\times(6\div2)+[10\times(6\div2)-2\times\frac{1}{2}\times(10\div2)\times(6\div2)]=30\,\text{cm}^2$；

Shaded area of figure (C) is $10\times\frac{1}{2}\times(10\div5)\times(6\div2)=30\,\text{cm}^2$；

Shaded area of figure (E) is $10\times\frac{1}{2}\times(6\div2)\times(10\div5)=30\,\text{cm}^2$.

Note that areas of figures (A), (B), (C) and (E) are all equal. Now we examine figure (D).

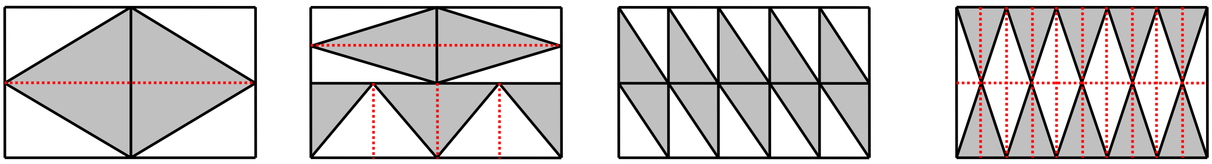
Note that the shaded parts of figure (D) are equally divided into two equal parts by a diagonal of the rectangle, with the sum of the areas equal.

Now, observe the diagonals of the 5 small triangles on the bottom half of the rectangle. We can see that the bases of these five triangles are all equal to 2 cm, and their heights are 1.2 cm, 2.4 cm, 3.6 cm, 4.8 cm and 6 cm, respectively, from left to right which is in the ratio 1: 2: 3: 4: 5. Therefore, the sum of the area of the five shaded triangles is $\frac{1}{2}\times(1.2+2.4+3.6+4.8+6)\times2=18\,\text{cm}^2$; therefore the total shaded area of the figure is 36 cm^2

Therefore, the figure that has the largest shaded region is figure (D).

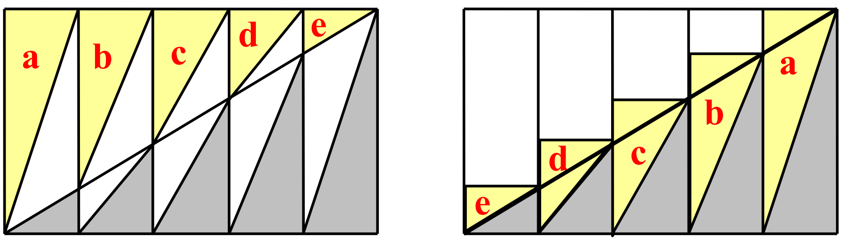
【Suggested Solution 2】

By drawing line segments in the figures (A), (B), (C), and (E) below, observe that the shaded areas of each figure is equal to the half of the total area of the rectangle.



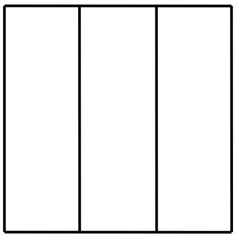
(A) (B) (C) (D)

While in figure (D), observe that the sum of the area is greater than half of the entire rectangle.



Answer：(D)

6. In the diagram below, a square of area 81 cm^2 is made up of three small identical rectangles . What is the perimeter of one small rectangle?



【Suggested Solution】

Since the area of the square is 81 cm^2, the side length of the square is 9 cm. Then the width of each small rectangle is 3 cm. Therefore the perimeter of each small rectangle is $2\times(3+9)=24$ cm.

Answer：24 cm

7. In adding up two numbers, Max has mistakenly read the tens digits '3' of the first number as '5' and the hundreds digit '9' of the second number as '6'. If Max got 2017 as the result, what should be the correct sum of the two numbers?

【Suggested Solution】

Mistakenly read the tens digit '3' as '5' causes an increase of 20 in the result.
Mistakenly read the hundreds digit '9' as '6' causes a decrease of 300 in the result.
Therefore the correct sum should be $2017-20+300=2297$.

Answer：2297

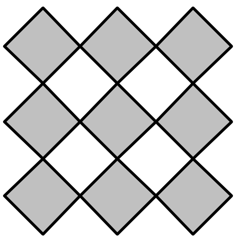
8. A hotel staff distributes four room keys to four travelers randomly. It is known that exactly two of the travelers get their correct room keys. How many different ways are there of distributing the keys to the four travelers?

【Suggested Solution】

Since 2 of the 4 travelers get their correct keys, then there is a total of $\frac{4\times3}{2}=6$ ways in doing this. Moreover, the 2 remaining travelers must be getting their correct keys, so there is only 1 way in doing such. So the total number of ways is $6\times1=6$ ways.

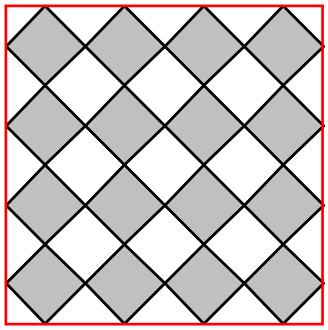
Answer：6 ways

9. Refer to the diagram below, where 9 black bricks and 4 white bricks are arranged alternately such that it will form a figure in which there are 5 bricks on its diagonals, and all the outer bricks are black.



Using the similar pattern stated above, if we want to form figures in which there are 7 bricks on its diagonals, how many black bricks are required?

【Suggested Solution 1】



Refer to the diagram on the right, you can see that the number of black bricks used is $1+2+3+4+3+2+1=4^2=16$.

【Suggested Solution 2】

When there are 7 bricks on its diagonals, each line has 4 black bricks, and will use 4 lines, therefore there is a total of $4\times4=16$ black bricks used.

【Suggested Solution 3】

Enclose the figure with a big square as shown. The total area of the white bricks is same as that of the black bricks. Suppose each brick of side length equals 1, the area of the big square is $\frac{1}{2}\times8\times8=32$. Therefore, the total number of black bricks is $\frac{1}{2}\times32=16$.

Answer：16 black bricks

10. The average weight of five cars *A*, *B*, *C*, *D* and *E* is 200 kg. If the average weight of cars *A*, *B* and *C* is 198 kg, while the average weight of cars *C*, *D* and *E* is 203 kg, what is the weight of car *C*, in kg?

【Suggested Solution】

The total weight of cars *A*, *B*, *C*, *D* and *E* is $200\times5=1000$ kg. The total weight of cars *A*, *B* and *C* is $198\times3=594$ kg, and the total weight of cars *C*, *D* and *E* is $203\times3=609$ kg. Therefore, the weight of *C* is $594+609-1000=203$ kg.

Answer：203 kg

11. Fill in each ◯ in the expression below with digits 1, 2, 3, 4, 5 and 6. Each digit can be used once and two 3-digit numbers are formed. What is the minimum difference of the expression below?

◯◯◯－◯◯◯= ______

【Suggested Solution】

All the digits are different. In order to make the difference minimum: the hundreds digits of the two numbers should be of difference equals 1. Next, consider the two-digit numbers formed by the tens and units digits of the minuend and the subtrahend respectively. The minuend should be made smallest while the subtrahend should be made the greatest.

So the minimum value of the expression is $412-365=47$.

Answer：47

12. Six pairs of integers are formed from the list of positive integers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11and 12. It is known that the sums of the five pairs are: 4, 6, 14, 20 and 21. What is the product of the remaining pair of numbers?

【Suggested Solution】

There is only one possible way to obtain the sum equals 4: $4=1+3$

It follows that there is only way to obtain the sum equals 6 : $6=2+4$

There are two possible ways to obtain the sum equals 21 and 20 respectively:

$21=9+12=10+11$; $20=8+12=9+11$

Since each number can be used once, we should have $21=10+11$ and $20=8+12$. From this, we can deduce that $9+5=14$, and the remaining pair is 6 and 7, and their product is 42.

Answer：42

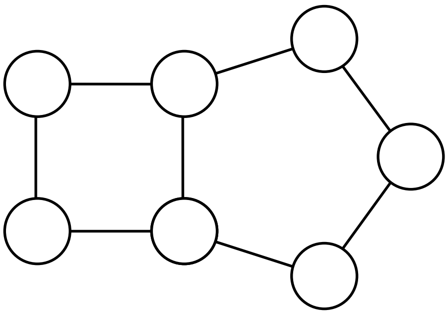
13. The sum of two three-digit numbers is 999. If the digits in the two three-digit numbers are all different without repetition, how many possible combinations are there?

【Suggested Solution】

Since all the digits are different, in adding the ones, tens and hundreds digit to get a sum of 9, there will not be any carry-overs. Therefore the only five possible pairs of digits to get a sum of 9 : (9, 0), (8, 1), (7, 2), (6, 3), (5, 4). Note that there are 4 ways to choose the hundreds digit ((9, 0) cannot be used because the hundreds digit cannot be 0), there are $4\times2=8$ ways to choose the tens digit, and $3\times2=6$ ways to choose the units digit. Therefore, there are a total number of $4\times(4\times2)\times(3\times2)=192$ ways that satisfies the condition.

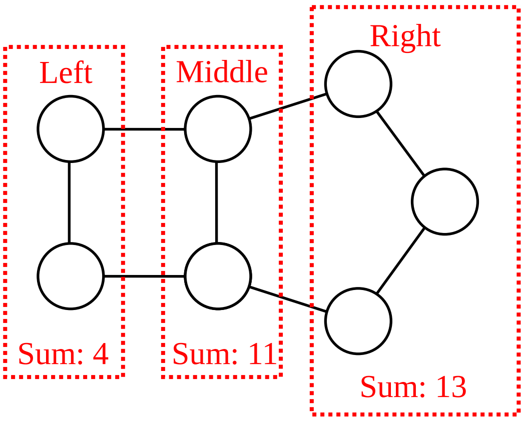
Answer：192 combinations

14. Fill in the circles with the numbers 1, 2, 3, 4, 5, 6 and 7. Each number can be used once without repetitions. The sum of the digits inside the circles at the four vertices of the square on the left is 15 and the sum of the digits inside the circles at the vertices of the regular pentagon on the right is 24. How many possible arrangements are there?



【Suggested Solution】

Since the sum of all the digits is $1+2+3+4+5+6+7=28$, we know that the sum of the 2 circles in the middle is $15+24-28=11$. (Refer to the figure below).



From this, we know that the only possible combination to get the sum of 4 for the left-most column is 1 and 3(which can be interchanged). Total of $2\times 1=2$ ways.
(5 points)

For the middle row, the only possible values to gett $11=5+6=4+7$, 2 ways and can be interchanged. Total of $2\times 2=4$ ways. (5 points)

For the right most column, possible value are 2, 4 and 7 (if 5 and 6 are used in the middle column) and 2, 5 and 6 (if 4 and 7 are used in the middle column), and we can arrange the numbers to be placed in the leftmost column in $3\times 2\times 1=6$ ways.
(5 points)

Therefore, the total number of possible arrangements is $2\times 4\times 6=48$ ways.
(5 points)

Answer：48 ways

15. A bag contains 2017 balls that are numbered from 1 to 2017. At least how many balls must be taken out so that among those balls, there will always be 3 balls, in which the sum of the numbers on the first two balls is the number on the third ball.

【Suggested Solution】

Since $2017=1008+1009<1009+1010$, suppose we take out 1009~2017, which is a total of 1009 numbers, we cannot ensure that there are three balls that will satisfy the given conditions.(10 points)

So suppose we take 1010 balls out. Suppose the largest number out of the 1010 balls is *M*, then the difference between *M* and the number of other balls taken out has 1009 different values, and both are less than 2017.(5 points) Since there are only 1007 balls that have not been taken out, at least one difference $M-x$ is the number y of the balls taken out, where *x* is the number of the removed ball. So *x*, *y*, $x+y=M$ are taken out of the ball number which satisfies the condition above. (5 points)

Answer：1010 balls