Solution Key to Second Round of IMAS 2018/2019
Middle Primary Division

1. What is the value of $100-97+94-91+88-85+\cdots+4-1$?

（A）45 （B）48 （C）51 （D）54 （E）57

【Solution 1】

$$100-97+94-91+88-85+\cdots+4-1=(100-97)+(94-91)+(88-85)+\cdots+(4-1)$$
$$=\underbrace{3+3+3+\cdots+3}_{17\text{ terms}}$$
$$=3\times17=51$$

Hence (C).

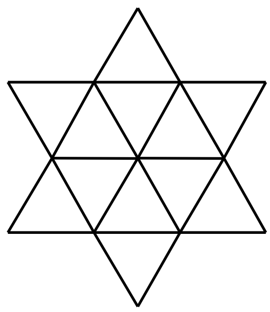
【Solution 2】

$$100-97+94-91+88-85+\cdots+4-1=(100+94+88+\cdots+4)-(97+91+85+\cdots+1)$$
$$=\frac{(100+4)\times17}{2}-\frac{(97+1)\times17}{2}$$
$$=(104-98)\times\frac{17}{2}$$
$$=6\times\frac{17}{2}=51$$

Hence (C).

Answer：(C)

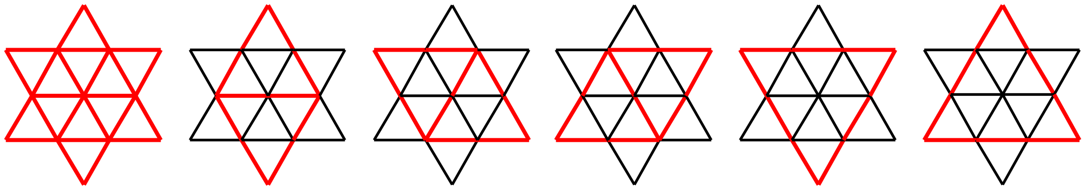
2. The figure below is formed by using 12 identical equilateral triangles. How many equilateral triangles of different sizes (and which are located in different places) are there?



（A）12 （B）14 （C）16 （D）18 （E）20

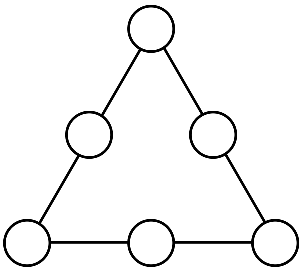
【Solution】

Let the area of a smallest equilateral triangle be 1 unit. There are 12 equilateral triangles of area 1. There are 6 equilateral triangles of area 4 and there are 2 equilateral triangles of area 9. Hence there are $12+6+2=20$ equilateral triangles at different positions. Hence (E).



Answer：(E)

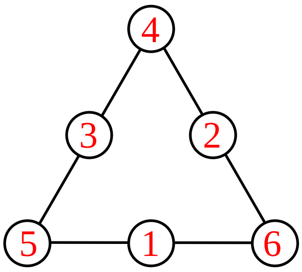
3. Place the numbers 1, 2, 3, 4, 5 and 6, without repetition, into the six circles in the figure below, where each circle should only have one number, such that the sum of the three numbers on each side of the triangle are all equal. What is the maximum possible value of this sum?



（A）9　　（B）10　　（C）11　　（D）12　　（E）13

【Solution】

To make required sum the largest, the sum of the numbers of the three sides must reach the maximum. Since the numbers at the vertices of the triangle are counted twice, so the largest numbers should be placed at the vertices, that is, 4, 5 and 6. Then sum of numbers on three sides is $1+2+3+4+5+6+4+5+6=36$, hence the sum of number on each side is then $36 \div 3=12$. Therefore, the numbers can be placed as follows. Hence (D).



Answer：（D）

4. Multiply a two-digit number by 3 then add 10 to it. Now, we swap the order of the two digits of the result. The resulting number is an integer among 95, 96, 97, 98 and 99. What is the original number?

（A）21　　（B）22　　（C）23　　（D）24　　（E）25

【Solution 1】

Working backwards, swap the digits of the following numbers: 95, 96, 97, 98, 99 and subtract by 10. The resulting numbers are 49, 59, 69, 79, 89, among which only 69 is a multiple of 3, so the original number is $69 \div 3=23$. Hence (C).

【Solution 2】

Multiply an integer by 3 and add 10, the resulting number has a remainder of 1 when it is divided by 3. Swapping the digits, the number also has remainder 1 when divided by 3. Among 95, 96, 97, 98, 99, only 97 has remainder 1 divided by 3. Thus, the original number is $(79-10) \div 3=23$. Hence (C).

Answer：（C）

5. If the month of January of some year has four Saturdays and five Sundays, then what day is January 17th of that particular year?

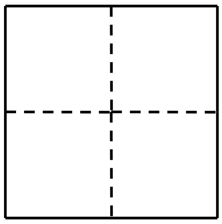
（A）Monday （B）Tuesday （C）Wednesday （D）Thursday （E）Friday

【Solution】

There are 31 days in January, which is three days more than 4 weeks. Since there are 4 Saturdays and 5 Sundays, the first day of the month is a Sunday and January 17th is a Tuesday. Hence (B)

Answer：（B）

6. A square has a perimeter 48 cm. Cut it into four identical small squares along the dashed lines as shown below. What is the sum of the perimeters of the four smaller squares?



【Solution 1】

The large square has perimeter 48 cm, hence its side length is 12 cm. Each small square has side length $12 \div 2 = 6$ cm and so perimeter $6 \times 4 = 24$ cm. Hence sum of the perimeter is $24 \times 4 = 96$ cm.

【Solution 2】

After cutting into 4 small squares, the total perimeter will be increase by double of the length of the dashed lines, which is equal to perimeter of the large square. Thus, the total perimeter of 4 small squares is $48 \times 2 = 96$ cm.

Answer：96 cm

7. Lily went shopping and bought three items from three different stores. She then noticed that whenever she was paying for an item, the money in her pocket was exactly five times the amount to be paid. After shopping, she noticed that she has $64 left in her pocket. How much money did she have before she went shopping?

【Solution 1】

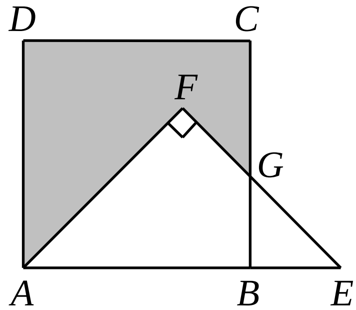
Working backwards, she had $64 \div 4 \times 5 = 80$ dollars before the third purchase. She had $80 \div 4 \times 5 = 100$ dollars before the second purchase. She had $100 \div 4 \times 5 = 125$ dollars before buying the first purchase.

【Solution 2】

Each time she bought, the money left is $\frac{4}{5}$ of the money before she bought. After three purchases, the money left was $\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{64}{125}$ of the money before the first purchase. So before the first purchase she had $64 \div \frac{64}{125} = 125$ dollars.

Answer：125 dollars

8. In the figure below, $ABCD$ is a square with side length of 10 cm and AFE is an isosceles right triangle with hypotenuse of length 14 cm, where E is on the extension of line AB. What is the area, in cm^2, of the shaded region?



【Solution 1】

From the given information, $BE = 14 - 10 = 4$ cm. Since triangle EBG is also an isosceles right triangle, its area is $\frac{1}{2} \times 4 \times 4 = 8$ cm^2. In isosceles right triangle AEF, the length of the height on hypotenuse AE is equal to the half of the length of AE, so the area of AEF is $\frac{1}{2} \times 14 \times 7 = 49$ cm^2. (or the area of isosceles right triangle AEF is equal to $\frac{1}{4} \times AE \times AE = \frac{1}{4} \times 14 \times 14 = 49$ cm^2.) Since the side length of square $ABCD$ is 10 cm, then the area of the shaded region is $10 \times 10 - (49 - 8) = 59$ cm^2.

【Solution 2】

Since $CG = CB - GB = CB - BE = CB - (AE - AB) = 10 - (14 - 10) = 6$ cm. Since triangle AEF is an isosceles right triangle, then $\angle FAE = 45^\circ$ and F is on the diagonal of the square, CFG is also an isosceles right triangle. The shaded area is equal to sum of area of triangle ACD and CFG, that is, $\frac{1}{2} \times 10 \times 10 + \frac{1}{4} \times 6 \times 6 = 59$ cm^2.

Answer：59 cm^2

9. There are a total of 40 students in a class. 23 of them are able to ride bikes, 33 of them are able to swim and 5 of them are unable to do either. How many students in this class are able to ride bikes but are not able to swim?

【Solution】

The number of students who are able to do at least one sport is $40 - 5 = 35$. Then the number of students who can do both sports is $23 + 33 - 35 = 21$. The number of students who can ride but not swim is then $23 - 21 = 2$ students.

Answer：2 students

10. A bridge is 1500 m long. A train passes through the bridge at a speed of 30 m per second. The train is 300 m long. How long, in seconds, does it take for the train to pass the bridge completely, starting from the time it entered the bridge?

【Solution】

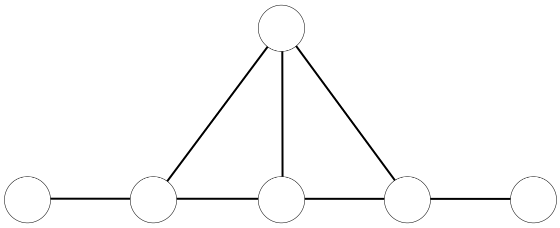
The train travels a distance of the bridge length and the train length, which is $1500 + 300 = 1800$ m. Hence it takes $1800 \div 30 = 60$ seconds for the train to leave the bridge completely.

Answer：60 seconds

【Note】

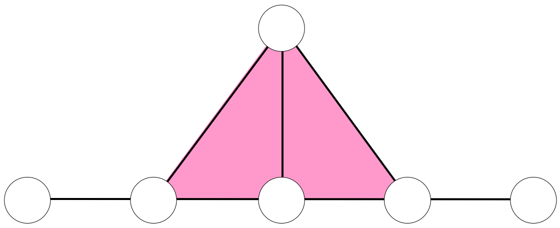
In a problem about a train going through a bridge, the time depends on the length of the bridge as well as on the length of train.

11. In the figure below, color each of the six circles into 4 colors: Red, Yellow, Blue and Black. Each circle should contain only one color, and any two circles connected by a line segment should have different colors. In how many different ways can we color the figure below? (Note: Coloring methods that are identical by a reflection of the figure are NOT considered the same)



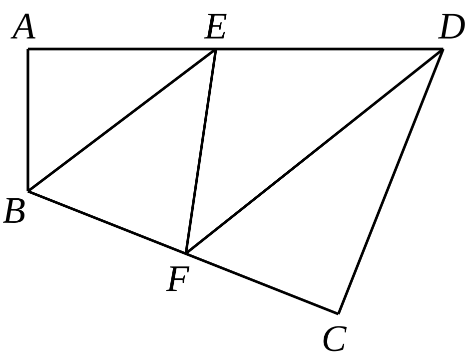
【Solution】

Focusing on the four circles in the triangle area. Color the six circles in a specific sequence. Firstly, there are 4 ways to color the top circle. Secondly, there are three ways to color the circle right under the top circle, which is not the same color of the first one. Thirdly, there are 2 ways to color the circle on the right vertex of the triangle, which is not the same color as the first circle and as the second circle. Fourthly, there are 2 ways to color the circle on the left vertex of the triangle. Fifthly and Sixthly, there are 3 ways each to color the last two circles on the bottom line, which has only one color excluded to use. Hence by multiplication principle, there are $4\times3\times2\times2\times3\times3=432$ ways to color all six circles.



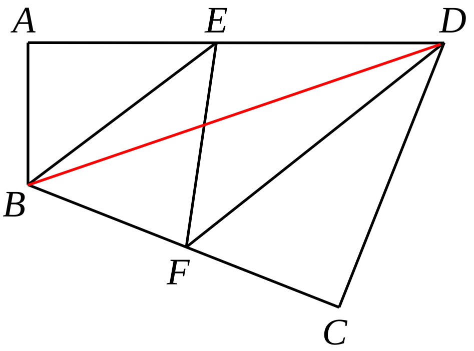
Answer：432 ways

12. In quadrilateral *ABCD*, $\angle BAD=\angle BCD=90°$. Points *E* and *F* are on sides *AD* and *BC* respectively and $AB=5\,\text{cm}$, $CD=10\,\text{cm}$, $DE=8\,\text{cm}$, $BF=6\,\text{cm}$, as shown in the figure below. If the area of triangle *BEF* is 4 cm^2 less than the area of triangle *DEF*. What is the area, in cm^2, of triangle *DEF*?



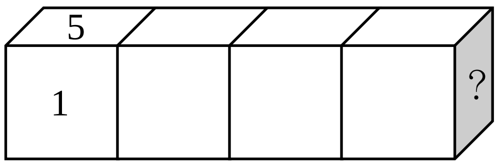
【Solution】

Connect BD. We know the area of triangle BDE is $\frac{1}{2}\times DE\times AB=\frac{1}{2}\times 8\times 5=20$ cm^2. The area of BFD is $\frac{1}{2}\times BF\times CD=\frac{1}{2}\times 6\times 10=30$ cm^2. Then the sum of areas of triangle BEF and DEF is $20+30=50\,\text{cm}^2$. Since the difference of area of triangle DEF and BEF is 4 cm^2, hence the area of triangle DEF is $(50+4)\div 2=27$ cm^2.



Answer：27 cm^2

13. The numbers 1, 2, 3, 4, 5 and 6 are written on the six faces of a unit cube without repetition. Each face contains one number and the sum of the numbers in every two opposite faces is 7. Put four such cubes side by side as shown in the figure below, such that sum of every two numbers of every two touched faces is 8. Find the number marked with "?" in the figure.



【Solution】

Since sum of numbers on opposite faces is 7, then 1 is opposite to 6, and 2 is opposite to 5, and 3 is opposite to 4. The right face of the first cube has number 3 or 4.
If it is 3, the left face of the second cube has number 5, right face has number 2, left face of the third cube has number 6, right face has 1, left face of the fourth cube has number 7, which is a contradiction.
If it is 4, the left face of the second cube has number 4, right face has number 3, left face of the third cube has number 5, right face has 2, left face of the fourth cube has number 6, right face has number 1, which is the place with mark "?" .

Answer：1

14. A mouse starts from the top left-most unit square marked with "*I*", follows a route to form the word "*IMAS2019*" by moving from one square to another square that share a common side. How many different routes of eight squares are there?

I	*M*	*A*	*S*	
M	*A*	*S*	*2*	*0*
A	*S*	*2*	*0*	*1*
S	*2*	*0*	*1*	*9*
	0	*1*	*9*	

【Solution】

In the following table, each square is filled with the number of routes reaching it. The number can be derived by recursion: each square is filled with sum of the numbers in adjacent squares with previous marks. From the table, it shows that the number of different routes with length eight is $34+34=68$.

1	1	1	1	
1	2	3	4	4
1	3	6	10	14
1	4	10	20	34
	4	14	34	

Answer： 68 routes

【Marking Schemes】

Find correct number of routes to squares with 2: $4+6+4=14$, 5 points.
Find correct number of routes to squares with 0: $4+10+10+4=28$, 5 points.
Find correct number of routes to squares with 1: $14+20+14=48$, 5 points
Find correct number of routes to squares with 9: $34+34=68$, 5 points.

15. An infinite sequence of numbers 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, … follows the pattern such that starting from the third number, each number is equal to the units digit of the sum of the two numbers in the sequence immediately preceding it. What is the 2019th number of the sequence?

【Solution 1】

This recurrence sequence follows a pattern when considering the remainders of numbers in the sequence divided by a fixed number. Since $10=2\times 5$, we observe the cases by dividing 2 and 5.

Considering remainders when each number of the sequence divided by 2, the pattern is

1, 0, 1, 1, 0, 1, 1, 0, 1, ….

The period is 3.

Considering remainders when each number of the sequence divided by 5, the pattern is

1, 2, 3, 0, 3, 3, 1, 4, 0, 4, 4, 3, 2, 0, 2, 2, 4, 1, **0**, 1, 1, 2, 3, 0, ….

The period is 20.

Since 2019 is divisible by 3, the remainder of the 2019th number when divided by 2 is the same as remainder of the third number, which is 1.

Since $2019=20\times 100+19$, the remainder of the 2019th number divided by 5 is the same as remainder of the 19th number, which is 0.

The 2019th number is a digit with remainder 1 when divided by 2 and remainder 0 when divided by 5, hence this number is 5.

【Solution 2】

The first seventy terms of the sequence are

1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0, 7, 7, 4, 1, 5, 6, 1, 7, 8, 5, 3, 8, 1, 9, 0, 9, 9, 8, 7, 5, 2, 7, 9, 6, **5**, 1, 6, 7, 3, 0, 3, 3, 6, 9, 5, 4, 9, 3, 2, 5, 7, 2, 9, 1, 0, 1, 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, ….

We can observe that it is a recurrence sequence with period 60. $2019 = 60 \times 33 + 39$, so the 2019th term is same as the 39th term, which is 5.

Answer：5

【**Note**】

The sequence is formed by the unit digits of Fibonacci sequence with period 60.

【Marking Schemes】

Find periodic pattern of parity of the sequence is 3, 5 points.

Find periodic pattern of the remainders divided by 5 of the sequence is 20, 5 points.

(Or find the sequence has period 60, 10 points.)

Find the correct answer without a reasoning, 10 points only.