Solution to
Eighth International Mathematics Assessment for Schools
Round 1 of Upper Primary Division

1. What is the simplified value of $2.5\times3.2\times0.76\times12.5$?

（A）0.76　（B）7.6　（C）76　（D）176　（E）760

【Suggested Solution】

$$2.5\times3.2\times0.76\times12.5=2.5\times4\times0.8\times12.5\times0.76$$
$$=10\times10\times0.76$$
$$=76$$

Therefore, the answer is（C）.

Answer: (C)

2. The purchase volume of apples in a hypermarket is 1560 kg, and it is three times that of pears. What is the total purchase amount of apples and pear, in kg, of that hypermarket?

（A）520　（B）2080　（C）2180　（D）4680　（E）6240

【Suggested Solution 1】

From the given information, the amount of pears purchased is $1560\div3=520\,\text{kg}$, then the total purchase amount of apples and pears in this hypermarket is $1560+520=2080\,\text{kg}$. Therefore, the answer is (B).

【Suggested Solution 2】

It is known total number of pears purchase is $\frac{1}{3}$ that of apples, that is; the total number of apples and pears purchase is $1+\frac{1}{3}=\frac{4}{3}$, so the total amount of apples and pears is $1560\times\frac{4}{3}=2080$ kg. Therefore, the answer is (B).

Answer: (B)

3. Which of the following statements is correct for the approximation of the "rounding method"?

（A）When representing approximations of numbers, 3.00 is the same as 3.0.

（B）The approximation 4.0 is as precise as 4.

（C）The result in approximation of 2.019×0.5 to the nearest thousandth is 1.009.

（D）If the approximate value of a certain number is 3, then this number must be less than 3.05.

（E）The approximation of repeating decimal number $2.\overline{019}=2.019019...$ to the

fifth place after the decimal point is 2.01902.

【Suggested Solution】

In option (A): When apply rounding off numbers, the range of original number for the approximate number 3.00 is $[2.995,\ 3.005)$;

When apply rounding off numbers, the range of original number for the approximate number 3.0 is $[2.95,\ 30.5)$.

Therefore, the accuracy of the two numbers is not the same, (A) is incorrect!

In option (B): When apply rounding off numbers, the range of original number for the approximate number 4.0 is $[3.95,\ 4.05)$;

When apply rounding off numbers, the range of original number for the approximate number 4 is $[3.5,\ 4.5)$.

Hence, the accuracy of the two numbers is not the same, and (B) is incorrect!

In option (C): Since $2.019\times 0.5=1.0095\approx 1.010$, (C) is incorrect!

In option (D): The approximate value of 3.1 after rounding is 3 and $3.1>3.05$, then (D) is incorrect!

In option (E): Observe that $2.\overline{019}=2.019019...\approx 2.01902$.

Therefore, the answer is (E).

Answer:（E）

4. Which of the following statements is correct?

（A）The quotient of the dividend divided by the divisor must be less than the dividend.

（B）The quotient obtained when the dividend is divided by the divisor must be greater than the dividend.

（C）The quotient obtained when a number is divided by 0.4 must be greater than this number.

（D）If both the dividend and divisor are greater than 0, then when the divisor is doubled, the quotient is doubled.

（E）If both the dividend and divisor are greater than 0, and if the quotient is less than the dividend, then the divisor must be greater than 1.

【Suggested Solution】

For option (A): $0.4\div 0.2=2>0.4$, so (A) is incorrect.

For option (B): $4\div 2=2<4$, so (B) is incorrect.

For option (C): $0\div 0.4=0$, it follows (C) is incorrect.

For option (D): $4\div 2=2$, $4\div 4=1$. Hence, (D) is incorrect.

For option (E): Represent the dividend as a, divisor as b, then the quotient is $\frac{a}{b}$.

When $\frac{a}{b} < a$, since a is a nonzero positive number, so when divided both sides of the inequality by a, we have $\frac{1}{b} < 1$ · that is; $b > 1$.

Therefore, the answer is (E). Answer: (E)

5. Use " < " to connect $12.5\dot{2}\dot{1}$, $12.\overline{521}$, $12.5\dot{2}\dot{1}$, 12.522.
Which of the following statement is correct?

(A) $12.52\dot{1} < 12.5\dot{2}\dot{1} < 12.\overline{521} < 12.522$ (B) $12.5\dot{2}\dot{1} < 12.52\dot{1} < 12.522 < 12.\overline{521}$

(C) $12.52\dot{1} < 12.5\dot{2}\dot{1} < 12.522 < 12.\overline{521}$ (D) $12.52\dot{1} < 12.\overline{521} < 12.5\dot{2}\dot{1} < 12.522$

(E) $12.5\dot{2}\dot{1} < 12.\overline{521} < 12.52\dot{1} < 12.522$

【Suggested Solution】

$$12.52\dot{1} = 12.521111...$$

$$12.5\dot{2}\dot{1} = 12.521212...$$

$$12.\overline{521} = 12.521521...$$

$$12.522 = 12.522$$

From the thousandth place, we know that 12.522 is the largest; from the ten thousandth place, $12.\overline{521}$ is the largest number, $12.\overline{521}$ is the second smallest number, and the smallest number is $12.52\dot{1}$. Therefore, the answer is (A).

Answer: (A)

6. The following table records the number of students in a certain primary six class in their mathematics mid-term exam.

Score	100 points	90~99 points	80~89 points	70~79 points	60~69 points	Below 60 points
No. of students	3	8	16	6	4	3

If the score of a student in this examination is more than 79 points, he/she will receive a gift. What is the percent rate of the students that will receive a gift in the mid-term exam? $\left(\text{Percent Rate} = \frac{\text{Number of students receiving gift}}{\text{Total number of students in the class}}\right)$

(A) 40% (B) 60% (C) 67.5% (D) 82.5% (E) 92.5%

【Suggested Solution】

From the given information, the total number of students is $3+8+16+6+4+3=40$.
The number of students whose score is more than 79 is $3+8+16=27$. Hence, the

percent rate is $\frac{27}{40}\times 100\% = 67.5\%$. Therefore, the answer is (C).

Answer: (C)

7. There are 110 meters of blue cloth, capable of making 22 sets of adult uniforms. After making 15 sets, the remaining cloth is just enough for 10 children's uniforms. How much less cloth, in meters, does a child uniform use than an adult uniform?

(A) 1 (B) 1.5 (C) 2 (D) 2.5 (E) 3

【Suggested Solution 1】

From the given information, the cloth needed for each adult uniform is $110\div 22=5$ meters, then 15 sets of adult uniforms need $5\times 15=75$ meters, so the remaining cloth material is $110-75=35$ meter, that is; the cloth of each set of children's uniforms needs $35\div 10=3.5$ meter. Thus, the cloth of each set of children's uniforms uses less $5-3.5=1.5$ meters than that of adults. Therefore, the answer is (B).

【Suggested Solution 2】

From the given information, the cloth needed for each adult uniform is $110\div 22=5$ meters. After 15 adult uniforms are made, the remaining 7 adult uniforms can be made into 10 children's uniforms. To make each children's uniform, only 0.7 times of the adult uniform cloth is needed, so the cloth of children uniform is used less $5\times(1-0.7)=1.5$ meters. Therefore, the answer is (B).

Answer: (B)

8. A classroom is 8 m long, 6 m wide and 4 m high. Jose wants to paint the ceiling and surrounding walls of the classroom, but the door and window with a total area of 22m^2 will not be painted. What is the total area that needs to be painted?

(A) 138 (B) 160 (C) 186 (D) 192 (E) 208

【Suggested Solution】

From the given information, the area of the ceiling is $8\times 6=48\text{m}^2$ and the total area of the surrounding walls is $2\times(4\times 8+4\times 6)=112$ m^2. Thus, the area to be painted is $48+112-22=138$ m^2. Therefore, the answer is (A).

Answer: (A)

9. A two-digit prime number has two distinct digits. When the two digits are swapped, one gets another prime number. How many such two-digit primes are there?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

【Suggested Solution】

Since the ones digit of a two-digit prime must be 1, 3, 7, or 9, it is possible to determine that the two digits of the two-digit prime number must be 1, 3, 19, 31, 37, 39, 71, 73, 79, 91, 93, 97. However, since 39, 91 and 93 are not prime numbers, 19 should be deleted, that is, only 13, 17, 31, 37, 71, 73, 79 and 97 are the two-digit prime number. Therefore, the answer is (D).

Answer:（D）

10. In a division of positive integers, the quotient is 24 and the remainder is 42. When the divisor takes the smallest possible value, what is the dividend?

（A）1008　（B）1032　（C）1050　（D）1074　（E）1242

【Suggested Solution】

Because the remainder is 42, then the smallest possible divisor must be 43, and the dividend is $24\times 43+42=1074$. Therefore, the answer is (D).

Answer:（D）

11. It is known that a three-digit number $\overline{2ab}$ is divisible by 6. What is the maximum possible value of $a+b$?

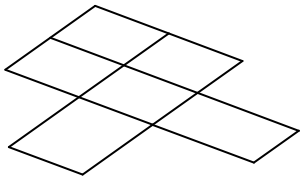
（A）14　（B）15　（C）16　（D）17　（E）18

【Suggested Solution】

If a number is divisible by 6, then the sum of all the digits of that number must be divisible by 3 and the ones digit must be even digit, it follows $2+a+b$ is divisible by 3. Since $a+b\le 9+9=18,\ 2+a+b\le 20,$ the largest possible value of $a+b$ can only be 16. This implies the possible three-digit number is 288. Therefore, the answer is (C).

Answer:（C）

12. How parallelograms with sides on the grid lines are there in the figure below?



（A）6　（B）8　（C）10　（D）13　（E）15

【Suggested Solution】

There are 6 small parallelograms of 1 single parallelogram.
There are 6 parallelograms consisting of two small parallelograms.
There are 2 parallelograms consisting of three small parallelograms.
There is one parallelogram consisting of four small parallelograms.
So, there are a total of $6+6+2+1=15$ parallelograms.

Therefore, the answer is (E).

Answer: (E)

13. The same number of white and black balls are placed in a box. In one operation, five white balls and three black balls were taken out from the box. After several operations, there were zero white balls and only 6 black balls left in the box. How many operations was performed?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

【Suggested Solution 1】

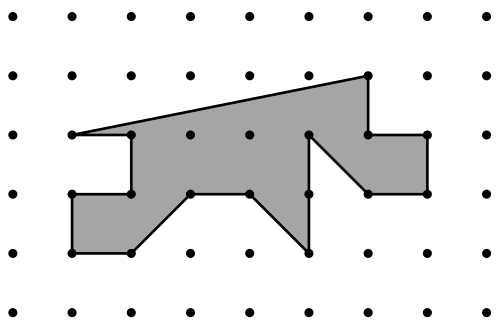
From each operation, it can be seen that the number of black balls in the box will be two more than the number of white balls. At the last operation, there were 6 more black balls than white balls in the box, so the total number of operations was $6 \div 2 = 3$ times. Therefore, the answer is (A).

【Suggested Solution 2】

Assuming there are x operations, there are $5x$ white balls and $3x+6$ black balls. Since the number of white balls and black balls is the same, then $5x = 3x+6$. So, $x = 3$. Therefore, the answer is (A).

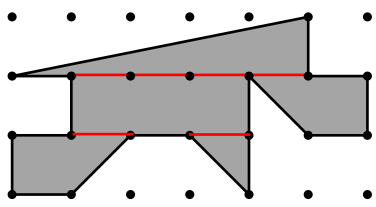
Answer: (A)

14. In the figure below, the distance between two adjacent vertices is 1 cm. What is the area of the shaded portion of the figure, in cm^2?



(A) 7.5 (B) 8 (C) 8.5 (D) 9 (E) 9.5

【Suggested Solution 1】

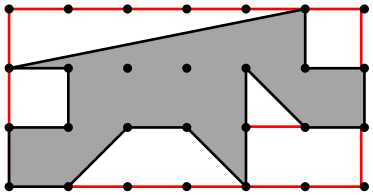


Cut the shaded portion into two right triangles, a rectangle and two trapezoids as shown in the figure, and the area of the shaded part can be computed as

$$\frac{5\times1}{2}+\frac{(1+2)\times1}{2}+\frac{1\times1}{2}+\frac{(1+2)\times1}{2}+1\times3=\frac{5}{2}+\frac{3}{2}+\frac{1}{2}+\frac{3}{2}+3=9\,\text{cm}^2.$$

Therefore, the answer is (D).

【Suggested Solution 2】



As shown in the figure, the area of the shaded part can be known by cutting out from a 3×6 rectangle: two right triangles, two small squares, a rectangle and a trapezoid:

$$3\times6-\frac{5\times1}{2}-1\times1-1\times1-\frac{1\times1}{2}-1\times2-\frac{(1+3)\times1}{2}=18-\frac{5}{2}-1-1-\frac{1}{2}-2-2=9\,\text{cm}^2.$$

Therefore, the answer is (D).

【Suggested Solution 3】

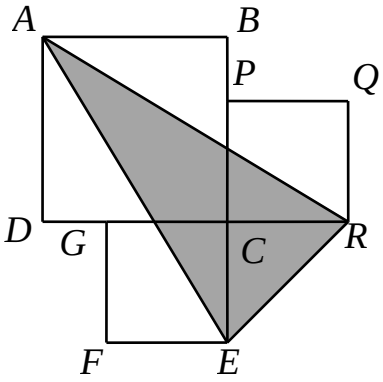
It can be seen that there are 16 points on side of the shaded region and 2 points inside the shadow region. Therefore, the area of the shaded region can be computed by the Pick Theorem as $2+\frac{16}{2}-1=9\,\text{cm}^2$. Therefore, the answer is (D).

Answer: (D)

15. The side length of square *ABCD* is 6 cm, while that of square *CEFG* and *CPQR* are both 4 cm, as shown in the figure. What is the area, in cm^2, of the shaded portion?

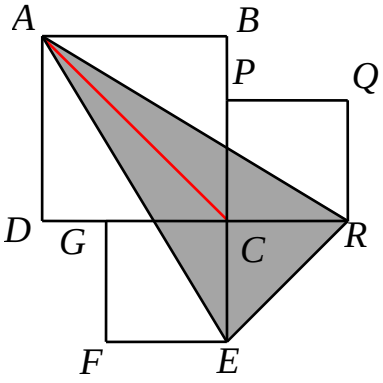
(A) 32 (B) 36 (C) 40

(D) 42 (E) 48

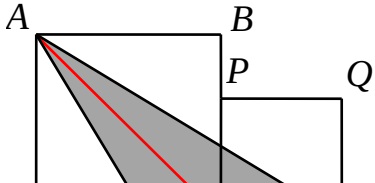


【Suggested Solution 1】

Connect *AC*, then the shaded portion will divided into three triangles, namely triangle *ACE*, *ARC* and *CRE*. So, the area of triangle *ACE* is $\frac{CE\times AB}{2}=\frac{4\times6}{2}=12\,\text{cm}^2$, the area of triangle *ARC* is $\frac{CR\times AD}{2}=\frac{4\times6}{2}=12\,\text{cm}^2$, the area of triangle *CRE* is $\frac{CE\times CR}{2}=\frac{4\times4}{2}=8\,\text{cm}^2$,. Hence, the area of the shaded portion is $12+12+8=32\,\text{cm}^2$. Therefore, the answer is (A).



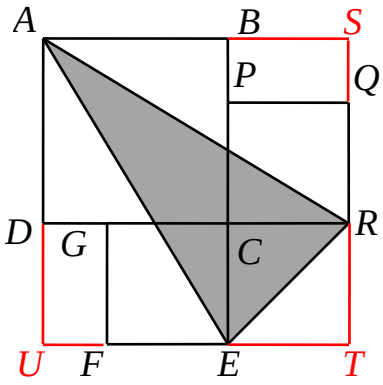
【Suggested Solution 2】



From the given information, we know each of the side length of square *CEFG* and square *CPQR* is 4 cm, $\angle ECR = 90°$, so that $RE = 4\sqrt{2}$ cm. Now make the perpendicular line of *ER* from point *A* and let the perpendicular foot be *T*. As shown in the figure and from property of symmetry $AR = AE$, then triangle *ARE* is an isosceles triangle. Therefore, it will pass point *C* and point *T* is the midpoint of *RE*. So that we have $AT = AC + CT = AC + \frac{RE}{2} = 6\sqrt{2} + 2\sqrt{2} = 8\sqrt{2}$ cm. Hence, the area of shaded portion is $\frac{AT \times ER}{2} = \frac{4\sqrt{2} \times 8\sqrt{2}}{2} = 32\,\text{cm}^2$.

Therefore, the answer is (A).

【Suggested Solution 3】



Extend *AB*, *AD*, *EF*, *QR* and let the extension of side *AB* intersect the extension of side *QR* at point *S*, the extension of side *QR* intersect the extension of side *EF* at point *T*, and the extension of side *EF* intersect the extension of side *AD* at point *U*, so that $\angle ASR = \angle RTE = \angle FUD = 90°$, then a large rectangle *ASTU*, as shown in the figure will be constructed. It follows the area of the triangle *ARE* can be determined by removing triangle *ASR*, *RTE* and *AEU* from the area of large rectangle *ASTU*. But $AS = AB + PQ = 6 + 4 = 10$ cm, $SR = AD = 6$ cm, $RT = TE = CE = 4$ cm, $EU = CD = 6$ cm, $AU = AD + DU = 6 + 4 = 10$ cm, then the area of shaded portion is

$$10 \times 10 - \frac{10 \times 6}{2} - \frac{4 \times 4}{2} - \frac{10 \times 6}{2} = 100 - 30 - 8 - 30 = 32\,\text{cm}^2.$$

Therefore, the answer is (A).

Answer: (A)

16. There are three kinds of lunch boxes in the school cafeteria, the prices are \$15, \$13 and \$11, respectively. At a certain day noon time, the total income of the cafeteria from selling lunch boxes was \$2019. How many lunch boxes might the cafeteria sold at noon time on that day?

(A) 105 (B) 130 (C) 155 (D) 185 (E) 205

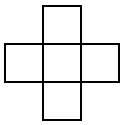
【Suggested Solution】

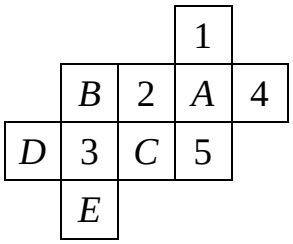
Since the price of each meal lunch box is an odd number, and the total income: 2019 is also an odd number, it follows the total number of lunch boxes sold must be an odd number also. Now, the most expensive lunch box is \$15, so the number of lunch boxes sold must be greater than $\frac{2019}{15} = 134\frac{4}{15}$, that is; at least 135 lunch boxes.

Since the cheapest lunch box is \$11, then the number of lunch boxes sold must be

less than $\frac{2019}{11}=183\frac{6}{11}$, that is; at most 183 lunch boxes. Only option (C) meet the given information of the problem. For example, three lunch boxes at \$15 per box are sold, 151 lunch boxes at \$13 per box are sold and one lunch box at \$11 per box is sold out, the total income is $3\times15+151\times13+1\times11=\$2019$. Therefore, the answer is (C).

Answer: (C)

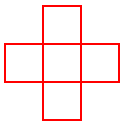
17. Each square in the figure below is filled with a positive integer so that the sum of each  (five squares) is divisible by 3.



What is the minimum value of $A+B+C+D+E$?

(A) 3 (B) 6 (C) 8 (D) 9 (E) 15

【Suggested Solution】

From the give figure, there are two  (of which there are 5 squares each), so we need to satisfy $1+2+4+5+A$, $3+B+C+D+E$ is divisible by 3.

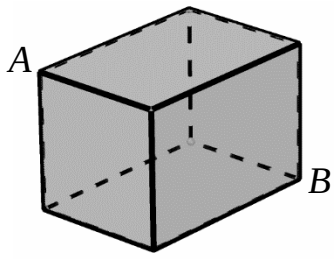
But $1+2+4+5+A=12+A$, then the minimum value of A is 3.

Since $3+B+C+D+E$ is divisible by 3, then $B+C+D+E$ is also divisible by 3 and we also have $B+C+D+E\geq1+1+1+1=4$, it follows the minimum value of $B+C+D+E$ is $6=1+1+1+3$.

Hence, the minimum sum of those numbers filled in these five blank squares must be $3+6=9$. Therefore, the answer is (D).

Answer: (D)

18. As shown in the figure below, an ant starting from vertex A of the cuboid needs to move along the edges to reach its destination, vertex B. If the ant can only pass through three edges, how many possible paths can the ant crawl to reach the destination?



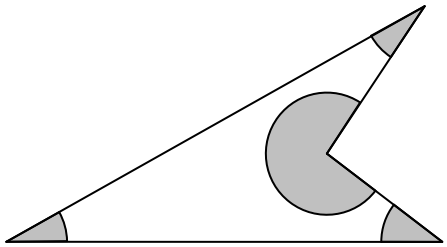
（A）3　　（B）6　　（C）9　　（D）12　　（E）15

【Suggested Solution】

The ant have a total of 3 edges starting from *A*, and after selecting the edge of the first one, the second edge has two choices and the third edge is also determined. So, there is a total of $3\times2=6$ different ways that the ant can travel from point *A* to *B*. Therefore, the answer is (B).

Answer：（B）

19. Given a concave quadrilateral, draw a sector with radius of 5 cm from each of the four vertices as shown below. What is the area, in cm, of these four sectors? (Assume $\pi=3.14$)



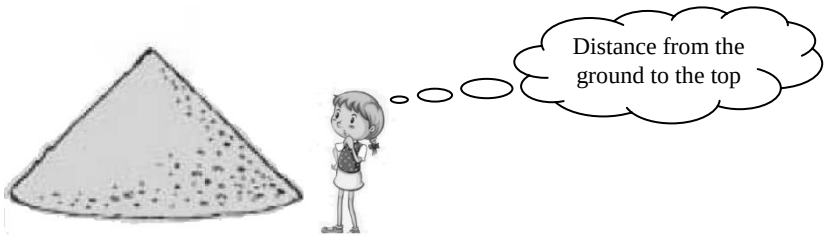
（A）39.25　（B）78.5　（C）157　（D）235.5　（E）Undetermined

【Suggested Solution】

Because the sum of all interior angles of the quadrilateral is 360°, so the four sectors can be cut to form a complete circle. So, the sum of areas of four sectors is $3.14\times5\times5=78.5\text{cm}^2$. Therefore, the answer is (B).

Answer:（B）

20. There is a conical wheat heap on the yard, which weighs about 10 385.55 kg. If the bottom circumference is 18.84 m, the weigh of the wheat is 735 kg per m^3. What is the approximate distance, in cm, from top of the heap to the ground? (Assume $\pi=3.14$)



（A）0.5　（B）0.8　（C）1　（D）1.2　（E）1.5

【Suggested Solution】

Since the perimeter of the base is a circle of 18.84 m ， so the radius of the circle is $\frac{18.84}{2\times 3.14}$ m. Assume the distance from the ground to the top of the conical is h m, then the volume of the conical is $\frac{1}{3}\times 3.14\times\left(\frac{18.84}{2\times 3.14}\right)^2\times h=\frac{18.84^2}{3\times 2^2\times 3.14}\times h=9.42h$ m^3, we know the weighs of wheat heap is $9.42h\times 735=10385.55$, that is $h=1.5$. Therefore, the answer is (E).

Answer:（E）

21. 34, 40 and 28 divided by a positive integer give the same remainder. What is the maximum possible value of this positive integer?

【Suggested Solution】

It can be seen that each of 34, 40 and 28 subtract the remainder will be a multiple of a certain positive integer. So this positive integer must be the common factor of $40-34=6$, $40-28=12$ and $34-28=6$. It follows that the possible maximum value of this positive integer is 6. Hence, the remainder obtained after dividing 34, 40 and 28 by 6 is 4.

Answer: 006

22. A pile of 54 cards contains a complete set of playing cards with two “ghost” cards. It is known that on top of the pile is “King heart”. If in each operation the top 4 cards are moved to the bottom without changing their order and direction. At least how many operations will it take such that “King heart” appears on the top again?

【Suggested Solution 1】

Because the least common multiple of 54 and 4 is 108, that after moving 4 cards per operation, a total of 108 cards will moved back to the original situation. Since we need to move 4 cards at a time, we must move at least $108\div 4=27$ times.

【Suggested Solution 2】

Because $54=4\times 13+2$, moving from top to bottom every 4 cards as a group, can be divided into 13 groups and the with 2 cards left over. So we conclude that after the 13 times of such operation, the "red heart K" will be located from top to bottom four cards on the third card; then perform another 13 times the same operation, this time the "red heart K" card located from top to bottom card on the fifth card. Then, perform another 1 time such operation, this time the "red heart K" card appear again at the top, that is; it needs to be operated at least $13+13+1=27$ times.

Answer: 027

23. There is a series of integers: 1, 1, 3, 5, 11, 21, 43, 85, 171, 341, 683, 1365, 2731, ... , this series has totally 109 numbers, and starting from the third number, each term is sum of the last number before it and twice the second last number before it. If we divide these 109 integers by 7, we get 109 remainders. What is

the sum of all these remainders?

【Suggested Solution】

Let us list down the first few remainders of these 109 integers when divided by 7 to:

1, 1, 3, 5, 4, 0, 1, 1, 3, 5, 4, 0, 1, 1, …,

At this point, it can be concluded that the remainders of 1, 1, 3, 5, 4, 0 are kept repeated to form a cycle of six numbers. Since $109=6\times18+1$, these six remainders have been repeated 18 times with one more remainder, and this remainder is the first of the six numbers. Because the sum if these six remainders $1+1+3+5+4+0=14$, the sum of these 109 remainders is $14\times18+1=253$.

Answer: 253

24. Suppose $a, b, c, d, e, f, g, h, i, j$ is an arrangement of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 with different letters representing different numbers. At most how many of the ten fractions $\frac{a}{b}, \frac{b}{c}, \frac{c}{d}, \frac{d}{e}, \frac{e}{f}, \frac{f}{g}, \frac{g}{h}, \frac{h}{i}, \frac{i}{j}, \frac{j}{a}$ can be reduced to integer values?

【Suggested Solution】

Since there are no multiples of 6, 7, 8, 9 and 10 in these ten numbers, then the value of a fraction can be an integer only if the denominator does not exceed 5, that is; there are at most 5 integers. On the other hand, desirable $a=8,\ b=4,\ c=2,\ d=1,$ $e=6,\ f=3,\ g=10,\ h=5,\ i=9,\ j=7.$ It follows $\frac{a}{b}=\frac{8}{4}=2,\ \frac{b}{c}=\frac{4}{2}=2,\ \frac{c}{d}=\frac{2}{1}=2,$ $\frac{e}{f}=\frac{6}{3}=2,\ \frac{g}{h}=\frac{10}{5}=2$ are integers.

Answer: 005

25. Arrange all positive integers less than 30 in a line, calculate the reciprocal of the product of every three successive numbers, then add the reciprocals to obtain *S*, that is;

$$S=\frac{1}{1\times2\times4}+\frac{1}{2\times4\times5}+\ldots+\frac{1}{26\times28\times29}.$$

What is the numerator of *S* reduced to the simplest fraction?

【Suggested Solution】

$$S=\frac{1}{1\times2\times4}+\frac{1}{2\times4\times5}+\ldots+\frac{1}{26\times28\times29}$$

$$3S=\frac{3}{1\times2\times4}+\frac{3}{2\times4\times5}+\ldots+\frac{3}{26\times28\times29}$$

$$=\frac{4-1}{1\times2\times4}+\frac{5-2}{2\times4\times5}+\ldots+\frac{29-26}{26\times28\times29}$$

$$=(\frac{1}{1\times2}-\frac{1}{2\times4})+(\frac{1}{2\times4}-\frac{1}{4\times5})+\ldots+(\frac{1}{26\times28}-\frac{1}{28\times29})$$

$$=\frac{1}{2}-\frac{1}{28\times29}$$

$$=\frac{405}{812}$$

Hence, $S=\frac{135}{812}$, we know that $135=5\times3^3$, which is relatively prime with 812.

So, the fraction $\frac{135}{812}$ is in simplest form. Therefore, the numerator is 135.

Answer: 135