2011 upper primary division second round test (solution)

1. The sides of the square 1600 cm^2 and 900 cm^2 are 40 cm and 30 cm respectively. Hence the area of the large square is 4900 cm^2, the shaded area is $4900-1600-900\times 3=600$ cm^2. Therefore, the shaded area required 6 pieces of square brick of 100 cm^2.

Answer: (A)

2. In the figure, there are $6^2-4=32$ squares. And $32\times 75\%=24$. There are 10 black squares and so 14 of the white squares must be painted black.

Answer: (D)

3. 【Solution 1】
The question told us that each male guest shakes hands 10 times and each female guest shakes hands 5 times. The total number of handshakes is 10×6+5×6=90. It takes two persons to shake hands, hence the total number of handshakes was counted twice. Therefore, there are 90÷2=45 handshakes among the 12 persons.
【Solution2】
The male and female guests shake hands 5×6=30 times and handshakes among male guests is 6×5÷2=15 times. So the number of handshakes among the 12 people is 30+15=45 times.

Answer: (B)

4. 21 days is equivalent to 504 hours, which is 3 weeks. Hence 500 hours from Tuesday 9am will be 5am on another Tuesday.

Answer: (B)

5. 【Solution 1】
As $C+D+E\le 9+9+9=27$, we need to consider the following three cases:
(i) If $C+D+E=22$, then $B=7$ and $A+F=4$; hence $A+10B+C+D+E+F=96$;
(ii) If $C+D+E=12$, then $B=8$ and $A+F=4$; hence $A+10B+C+D+E+F=96$;
(iii) If $C+D+E=2$, then $B=9$ and $A+F=4$; hence $A+10B+C+D+E+F=96$;
So $A+10B+C+D+E+F=96$.
【Solution 2】
From $(100A+20+E)+(100+10B+D)+(100F+20+C)=632$, we have
$$100A+100F+10B+C+D+E=492$$
$$99(A+F)+(A+10B+C+D+E+F)=492$$
$$A+10B+C+D+E+F=492-99(A+F)$$
Hence $A+10B+C+D+E+F=492$, 393, 294, 195 or 96.
Since $A+10B+C+D+E+F\le 9+90+9+9+9+9=135$, it implies that $A+10B+C+D+E+F=96$

Answer: (C)

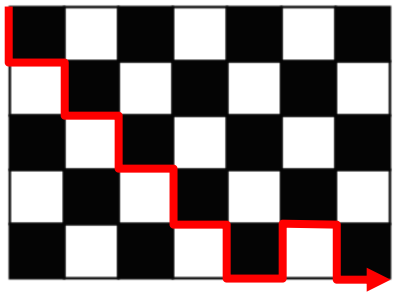
6. The number of passengers board the bus before it was full is $7-2=5$. This is equivalent to one-sixth of the seats. So the number of seats available in the bus is $5\div\frac{1}{6}=30$.

Answer: 30

7. In order to maximize the product of two 2-digit number, the larger number must be placed in the tenth-places, and keeping the difference of the two 2-digits numbers a minimum. Hence the maximum product is $63\times54=3402$. To obtain the minimum product of two two-digit number, the smaller number must be placed in tenth-places, and keeping the difference of the two 2-digit numbers a maximum. Hence the minimum product is $35\times46=1610$. The difference of the two products is 1792.

Answer: 1792

8. In order to achieve a shortest distance from point A to point B, the ant should avoid crawling left or upward. However, to ensure that the left hand side covered is always a black square, the ant could not crawl down or right for two consecutive squares. Every square covered vertically will be followed by a squared covered horizontally. Similarly, every square covered horizontally will be followed by a squared covered vertically. From point A to pint B, the ants need to crawl 7 squares horizontally and 7 squares vertically. So the ants need to crawl upward at least once. The thick line in the following figure shows one of the possible routes for the ants to cover a shortest distance from A to B, which is 14 cm.



Answer: 14cm

9. Let the great common divisor of a and b be d, and $a = dm$, and $b = dn$; m and n are relatively prime. The L.C.M of a and b is then dmn. From the information of the question, we have

$$dm+dn=d(m+n)=432 \text{ and } dmn+d= d(mn+1)=7776.$$

The two equations give us

$$mn+1=18(m+n) \text{ or } mn-18(m+n)+1=0.$$

Hence $(m-18)(n-18) = 18^2-1 = 17\times19$. Assume $m > n$, we have $(m-18) =17\times19=323$ and $n-18=1$; or $(m-18) =19$ and $(n-18) =17$. If m=341 and n =19; then $m+n$=360 and there is no positive integer d satisfying $d(m+n) = 432$. So this set of solution is not the required solution. If $m = 37$ and $n = 35$, then $d=432/(37+35)=6$. Hence $a = dm = 222$ and $b = dn = 210$. The product of a and b is 46620

Answer: 46620

10. Let x be the weight of the turnip just harvested, and then the turnip contain $0.9x$ kg of water. After an hour under the sun, the water content decreases by 10%, that is a decrease of $10\%\times0.9x= 0.09x$ kg. The weight of the turnip is $x-0.09x=0.91x$ kg, and the percentage of water in turnip is

$$\left(0.9-0.09\right)x \div (0.91x) \times 100\% = \frac{81}{91} \times 100\% \approx 89.01\%$$

Answer: 89.01%

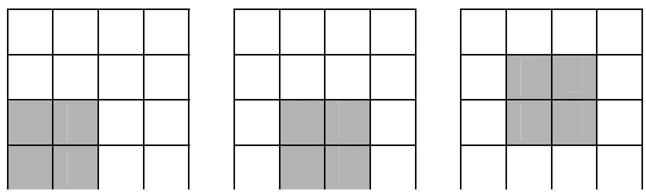
11. If the surface of the wood and the mouth of the bottle are in the same horizontal level, the height of water inside the bottle is $20-6\times\frac{1}{3}=18$ cm. the volume of the water is $3.14\times5^2\times18-6^3\times\frac{2}{3}=1269$ cm^3.

Answer: 1269 cm^3

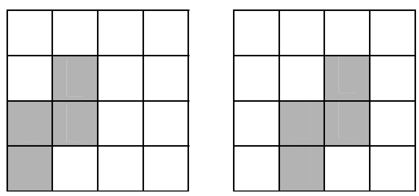
12. Let a, b and c be the number of frames made respectively for equilateral triangle, square and regular pentagon by using 100 sticks. In order that at least one of the three kinds of frames was made, we have the condition 100 – 3 – 4 – 5 = 88. If we want to have as many frames made as possible, then more frame of equilateral triangle should be made, and the maximum number of frame made is 29+3=32. In order to have as few frames made as possible, then more regular pentagons should be made, and the minimum number of frames made is 18+3=21. We will prove that the number of frames made could be any integer between 21 and 32. Let the total number of the frames made be $21+x$, where $1\le x\le 10$. As $21+x = a+b+c$, then $100=3(a+b+c)+b+2c=3(21+x)+b+2c$. Simplified, we have $b+2c=37-3x$. When $1\le x\le 10$, b and c could be positive integers, and $b+c < 21+x$. Hence the total number of frames is a positive integer between 21 and 32. Therefore, there are 12 possible values for the total number of the frames made.

Answer: 12

13. According to the symmetric property of square, there are three different settings for block (b) to be placed in a 4×4 square. Block (a) cannot be place in the third setting. If block (a) is put in the two other settings, it will lead to an empty region where no blocks could be placed. Hence, block (a) and block (b) could not be selected simultaneously. Similarly, block (b) and (e) cannot be selected simultaneously.



According to the symmetry property of a square, there are two settings for block (e) to be placed in a 4×4 square. Hence, a 4×4 square could not be formed by using block (e) alone. If block (e) is selected, block (d) should also be selected simultaneously.

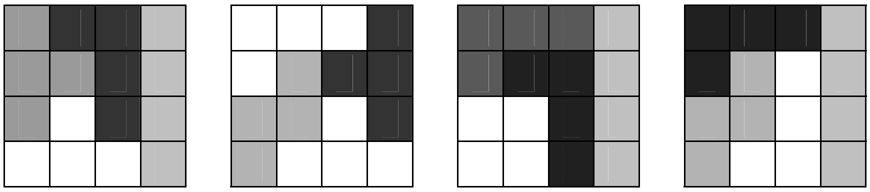


(1) Only one kind of blocks is used: The first four kinds of blocks can form a 4×4 square (4 pieces each). There are has four choices.

(2) Two kinds of blocks are used together: Block (a) could not be used together

with other blocks to form a 4×4 square. Block (e) could only be used together with block (d) to form a 4×4 square. In the remaining three blocks, any two types of blocks could be put together to form a 4×4 square. Hence there are four choices.

(3) Three kinds of blocks are used together: Block (a) and (b) could not be selected at the same time; similarly blocks (b) and (e) could not be selected simultaneously. Hence, there are four choices. They are (a)(c)(d), (a)(d)(e), (b)(c)(d) and (c)(d)(e). The arrangements of blocks are as follows:



(a)(c)(d) (a)(d)(e) (b)(c)(d) (c)(d)(e)

(4) Four kinds of blocks are used together: As block (a) and (b) could not be selected simultaneously, similarly block (b) and (e) could not be selected simultaneously. And block (a) (b) (c) (d) could not form a 4x4 square. Hence, so such arrangement exists.

To conclude, there are 12 choices to form a 4×4 square

Answer：12

14. There is no five-digit "Magic Number". Suppose not, let N be the five-digit "Magic Number". The five digits of the "Magic Number" is arranged as a, b, c, d and e according to their magnitude in descending order. From the condition, $N = \overline{abcde} - \overline{edcba}$. According to the magnitude of a, b, c, d and e, the order of the five digits of N's is $a-e, b-d-1$, 9, $9+d-b$ and $10+e-a$. **(5 points)** Noted that 9 is the largest number, and obviously $a = 9$. As $9+d-b$ and $10+e-a$ are both larger than e, and $a-e=9-e \neq e$; we have $b-d-1=e$. **(5 points)**

Then $9+d-b=8-e$, and $10+e-a=e+1$. Hence the order of the five digits of N is $9-e$, e, e+1 and $8-e$. **(5 points)**

According to the magnitude of the value of a, b, c, d and e, we have $d=e+1$ and $b=9-e$. And from $b-d-1=e$, we have $b=2e+2$, and so $9-e=2e+2$. And And obviously such value of e doe not exist. Therefore, no such five-digit "Magic Number" exists. **(5 points)**

Answer: No such number exists

The Rubric for the marking (points given)

0 point：	Conclusion given, but without any explanation.
5 point：	Using proof by contradiction, using the letters a, b, c, d, e (or other letters expression) and list them according to the order of magnitude, and express N based on the magnitude of the numbers.
5 point：	Express each number digit of N in terms of a, b, c, d and e.
5 point：	Obtain the relationship among the numbers a, b, c, d and e; using one of them to express the number digit in N.

5 point： Using each of the number digits in N to prove the conclusion.

15.【Proof 1】

We estimate an upper limit for the number of stamps being cut. It is easy to cut out 10 pieces of stamp (two stamps in the middle of every line). Cutting 11 pieces of stamp is also possible (as shown on the right figure). **(10 points)**

	X		X		X	
		X		X		
	X				X	
		X		X		
	X				X	

We will then prove that 11 is the maximum number of stamp that could be cut.

Suppose 12 pieces of stamp could be cut. Let the length of each stamp be one unit. If 12 pieces of stamp were cut, by conditions 1 and 2, the "perimeter" of the sheet of remaining stamps is 28+4×12=76. On the other hand, we can tackle the question as if we are going to stick the stamps back on a square frame of stamps. The perimeter of a square frame of stamps is (7+5)×4=48, and we are required to stick back $5^2-12=13$ pieces of stamp. Each time a stamp is stick to the frame, the total perimeter will increase by 2. Hence the maximum perimeter of the stamps put back is $48+2\times13=74<76$. This is a contradiction. Therefore, Leon could not cut more than 12 pieces of stamp under such conditions. **(10 points)**

Answer: 11 pieces

The Rubric for the marking (points given)

10 point： Only the answer and the diagram are given.

10 point： Proof of the conclusion is given.

【Proof 2】

According to condition (1), at most 25 stamps could be cut. And according to condition (2), at most 13 stamps could be cut (as shown in figure (a)) **(5 points).**

	X		X		X	
		X		X		
	X		X		X	
		X		X		
	X		X		X	

(a)

		X		X		
	X		X		X	
		X		X		

(b)

	X		X		X	
		X		X		
	X				X	
		X		X		
	X				X	

(c)

In the original large sheet fo stamps, at most 3 stamps could be cut in each row. If 3 stamps is cut in one row, then at most 2 stamps could be cut on its adjunct row, with proper location of cutting (as shown in figure (b)) . Assume that 12 stamps could be cut, then at least 2 of the rows having 3 stamps being cut. If 3 stamps was cut from the third row (or fourth, fifth row) , then condition (3) is not satisfied (as shown in figure (b)). Hence the cases for cutting 2 rows of 3 stamps each could only be for the 2^{nd} and the 6^{th} row. It is easy to see that such cases do not satisfy conditions (3). **(5 points)**

Hence at most 11 stamps could be cut, as shown in figure (c) **(10 points).**

The Rubric for the marking (points given)

5 point： Provide the answer of at most 13 stamps were cut, according to conditions (1) and (2).

10 point： Provide the answer of at most 13 stamps were cut, according to conditions (1) and (2), and prove that at most 11 stamps were cut.

【Proof 3】

At most 11 stamps were cut, as shown in figure (a) **(10 pints).**

Next show that no more stamps could be cut.

According to condition (1), at most 25 stamps could be cut. And according to condition (2), at most 13 stamps could be cut. There is only one way, as shown in figure (b). **(5 points).**

	X		X		X	
		X		X		
	X				X	
		X		X		
	X				X	

(a)

	X		X		X	
		X		X		
	X		X		X	
		X		X		
	X		X		X	

(b)

		X		X		
	X		X		X	
		X		X		
	X		X		X	
		X		X		

(c)

Obviously, figure (b) does not satisfy condition (3).

Next we show that the case of 12 stamps being cut does not exist.

There are two cases to cut 12 stamps In accordance to conditions (1) and (2). The first case: to stick one stamp in figure (b). The second case: as shown in figure (c).

It is easy to show that to stick one stamp in figure (b) could never satisfy condition (3).

Obviously figure (c) could not satisfy condition (3).

Hence 12 stamps could not be cut. **(5 points)**

The Rubric for the marking (points given)

10 point： Only the answer and the diagram are given.

5 point： To show that there are two cases to have more than 11 stamps, if in accordance with condition (1) and condition (2). The cases are 13 stamps and 12 stamps.

10 point： To show that there are two cases to have more than 11 stamps, if in accordance with condition (1) and condition (2). The cases are 13 stamps and 12 stamps. And show that cutting 13 stamps and 12 stamps could not meet the condition, and conclude that at most 11 stamps could be cut.