Solution to
Fifth International Mathematics Assessment for Schools
Round 1 of Upper Division

1. What is the value of $162+1620+6201+2016$?

（A）9459　（B）9639　（C）9819　（D）9999　（E）10089

【Solution】

$162+1620+6201+2016=9000+900+90+9=9999$.

Answer：（D）

2. Which of the following five expression is correct?

（A）$1.2\times3.4=12\times3.4$　（B）$0.98\times0.99>0.99$　（C）$\frac{1}{2}-\frac{1}{3}<\frac{1}{3}-\frac{1}{4}$

（D）$10.4\times0.1<1.04$　（E）$1.1\times1.1>1.1$

【Solution】

In (A)： 12×3.4 is ten times 1.2×3.4, so the equation is incorrect.

In (B)： Since $0.98<1$ and when a positive number multiply a positive number less than 1, the product will become smaller, so the inequality is incorrect.

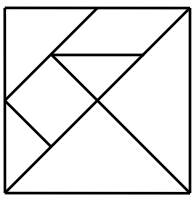
In (C)： $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}>\frac{1}{12}=\frac{1}{3}-\frac{1}{4}$, so the inequality is incorrect.

In (D)： A number multiply 0.1 can be viewed as moving its decimal point to one digit left. After moving the decimal point of 10.4 to one digit left, the number becomes 1.04. So the inequality is incorrect.

In (E)： $1.1>1$ when a positive number multiply a positive number greater than 1 the product will become larger. So the equation is correct.

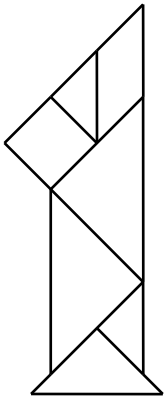
Answer：（E）

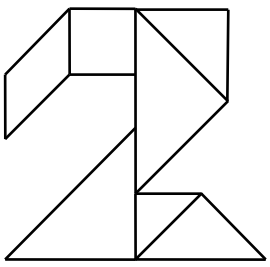
3. The diagram below shows the seven pieces in the classic Chinese puzzle called Tangram.

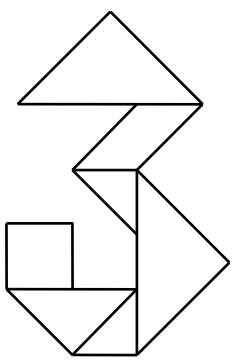


Which of the following five figures is not composed with a set of Tangram pieces?

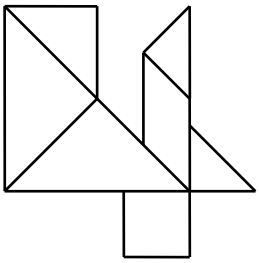
（A）　（B）　（C）

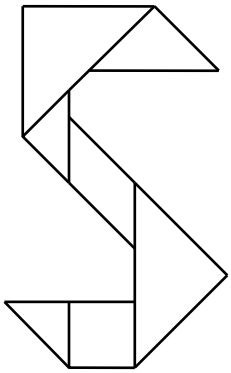






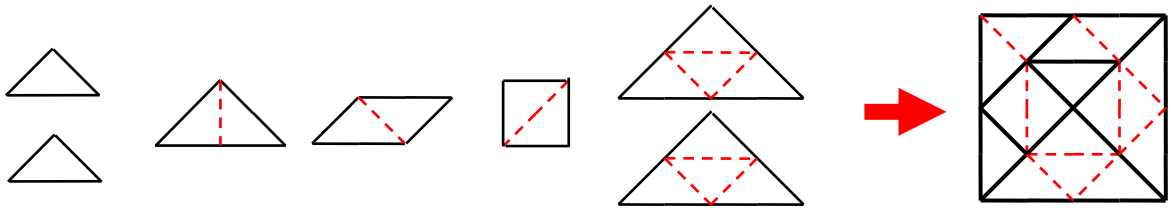
（D）　　（E）



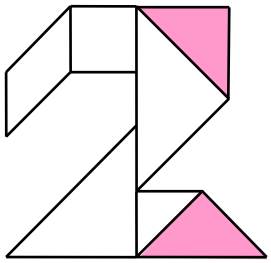


【Solution】

A set of tangram pieces includes two unit isosceles right-angled triangles, one 2-unit isosceles right-angled triangle, one 2-unit parallelogram, one 2-unit square and two 4-unit isosceles right-angled triangles, as shown below:



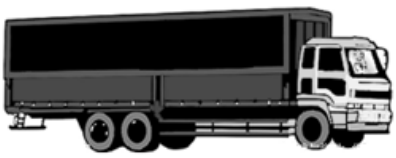
In (B), there is one less 1-unit isosceles right-angled triangle and one more 2-unit isosceles right-angled triangle. So (B) is not composed with a set of Tangram pieces.



Answer：（B）

4. A large truck can carry 6.3 tons and costs 1000 dollars to rent. A small truck can carry 2.1 tons and costs 400 dollars to rent. To transport 12.6 tons, how much cheaper if only large trucks are rented, compared with only small trucks are rented?

（A）100　　（B）200　　（C）250　　（D）350　　（E）400

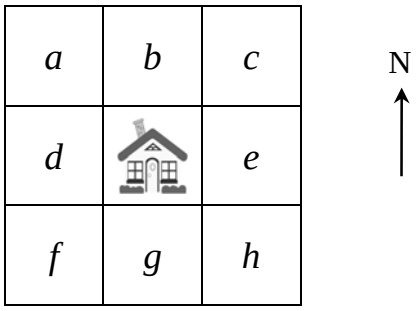


【Solution】

If all of 12.6 tons are carried by large trucks, then we need $12.6 \div 6.3 = 2$ large trucks and must pay $2 \times 1000 = 2000$ dollars. If all of 12.6 tons are carried by small trucks, then we need $12.6 \div 2.1 = 6$ small trucks and must pay $6 \times 400 = 2400$ dollars. The fare of only large trucks are rented is $2400 - 2000 = 400$ dollars cheaper than only small trucks are rented.

Answer：（E）

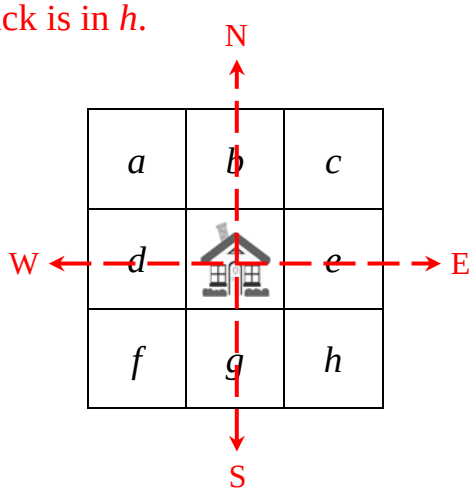
5. Mick is in one of the eight squares round a house, and the house is to his north-west. On which square is Mick?



（A）a　　（B）c　　（C）f　　（D）h　　（E）d

【Solution】

The house is at Mick's north-west, hence Mick is at the house's south-east. Shown as the figure below. So Mick is in h.



Answer：（D）

6. The table below summarizes the results of a test in a certain class. What is the total score of this class?

Summary of the results of a test			
No. of students	The highest score	The lowest score	The average score
42	100	16	84.5

（A）672　　（B）3528　　（C）3549　　（D）4200　　（E）4872

【Solution】

The total score of this class is $84.5\times 42=3549$.

Answer：（C）

7. How many positive common divisors do 192 and 120 have?

（A）1　　（B）2　　（C）6　　（D）8　　（E）10

【Solution】

Observe that $192=2^6\times 3$ and $120=2^3\times 3\times 5$, so the largest common divisor is $2^3\times 3=24$. Hence all of the divisors of 24 are the common divisor of 192 and 120. Since $24=2^3\times 3$ has $(3+1)\times(1+1)=8$ divisors, 192 and 120 also have 8 common divisors.

Answer：（D）

8. In reading a story book, Lance reads one page more each day than the preceding day. On the fourth day, he reads 39 pages. After 9 days, he still has 48 pages to go. How many pages are there in this book?

（A）351　　（B）399　　（C）360　　（D）408　　（E）432

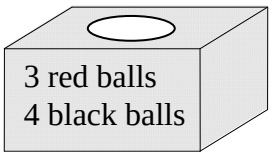
【Solution】

Since Lance reads 39 pages on the fourth day, he reads $39-3=36$ pages on the first day. So he has read 36, 37, 38, 39, 40, 41, 42, 43 and 44 pages in the first nine days. Thus there are $36+37+38+39+40+41+42+43+44+48=408$ pages in this book.

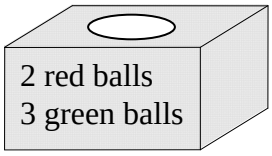
Answer：（D）

9. The contents of the five boxes are labeled. A ball is drawn at random from each box. From which box is the drawn ball most likely to be red?

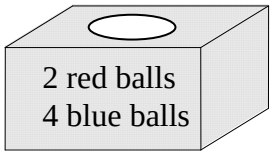
（A）



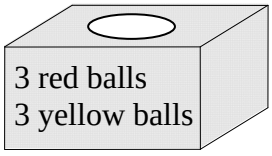
（B）



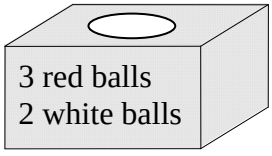
（C）



（D）



（E）



【Solution】

For the box in (E), the number of red balls is greater than the number of white balls, so the possibility to take red ball is greater then half. For the other options, the number of red balls is less than or equal to the number of balls of other colors. So the ball drawn from the box in (E) is most likely to be red.

Answer：（E）

10. Gasoline costs 6 dollars per liter. A car uses up 8 liters for every 100 km. What is the largest integral number of km that can be covered with 200 dollars worth of gasoline?

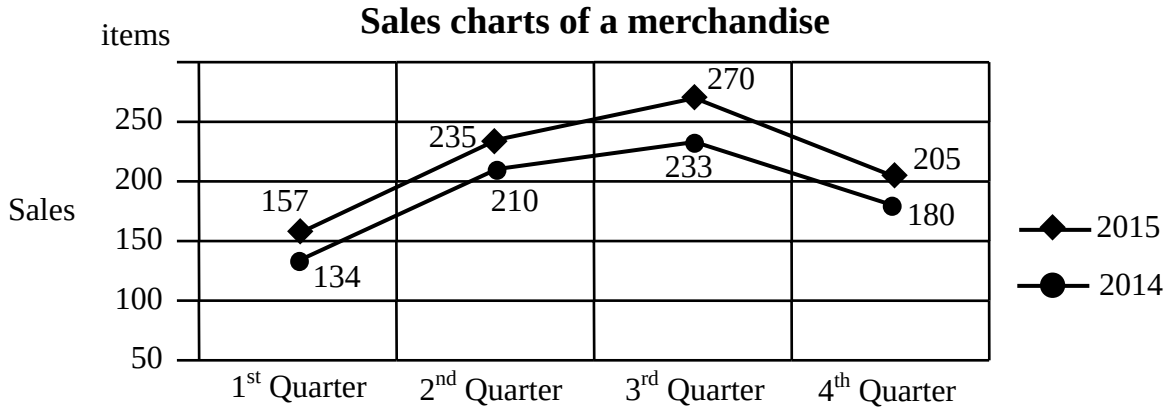
（A）416　　（B）417　　（C）418　　（D）419　　（E）420

【Solution】

The car can be added $200\div 6=\frac{100}{3}$ liters. Since the car uses up 8 liters for every 100 km, it can run $\frac{100}{3}\div 8\times 100=\frac{10000}{24}=416\frac{2}{3}$ km at most. The largest integral number of km that can be covered with 200 dollars worth of gasoline is 416.

Answer：（A）

11. The chart below shows the sale figures of a certain merchandise in 2014 and 2015 by the season. How many more items were sold in 2015 than in 2014?



（A）23　　（B）48　　（C）85　　（D）90　　（E）110

【Solution 1】

$157+235+270+205=867$ items were sold in 2014 and $134+210+233+180=757$ in 2015. So $867-757=110$ more items were sold in 2015 than in 2014.

【Solution 2】

$157-134=23$ more items were sold in the first quarter of 2015 than in the first quarter of 2014, $235-210=25$ more items were sold in the second quarter of 2015 than in the second quarter of 2014, $270-233=37$ more items were sold in the third quarter of 2015 than in the third quarter of 2014 and $205-180=25$ more items were sold in the fourth quarter of 2015 than in the fourth quarter of 2014. So $23+25+37+25=110$ more items were sold in 2015 than in 2014.

Answer：（E）

12. Fanny has 20 coins each worth 5 pence. Trading some of them for coins each worth 2 pence, she ends up with 32 coins. Then she trades some more 5-pence coins for coins each worth 1 penny, and now she has 56 coins. How many 5-pence coin does Fanny still have?

（A）5　　（B）6　　（C）7　　（D）8　　（E）9

【Solution】

Two 5-pence coins can be exchanged with five 2-pence coins and hence there are 3 more coins after such an exchange. Since Fanny has $32-20=12$ more coins after the first trading, she has taken $12\div3\times2=8$ 5-pence coins for the first trading. Similarly, one 5-pence coin can be exchanged with five 1-penny coins and hence there are 4 more coins after such an exchange. After second trading, Fanny has $56-32=24$ more coins, so she has taken $24\div4=6$ 5-pence coins for the second trading. Thus she still have $20-14=6$ 5-pence coins.

Answer：（B）

13. Every pair of the numbers from 1 to n is added, and there are 215 different sums. What is the value of n?

（A）100　（B）105　（C）108　（D）109　（E）215

【Solution】

The largest sum of each pair of the numbers is $n+(n-1)=2n-1$ and the smallest sum of each pair of the numbers is $1+2=3$, so the possible values of the sum of each pair of the numbers are all of the integers from 3 to $2n-1$. There are $2n-3$ different sums. So $2n-3=215$, i.e., $n=109$.

Answer：（D）

14. In a library, 12.1% of the books are fictions. After 1800 fictions and 2400 non-fictions go on loan, only 12% of the remaining books are fictions. How many books are there in the library initially?

（A）1296000　（B）1582200　（C）1800000　（D）1586400　（E）1291800

【Solution】

Assume that there are x books in the library now. So the number of the original fictions is

$$x\times 12\%+1800=(x+1800+2400)\times 12.1\%$$

Hence $x=1291800$. Thus there are $1291800+1800+2400=1296000$ books in the library initially.

Answer：（A）

15. How many two-digit numbers are there such that when 304 is divided by the two-digit number leaving the remainder 24?

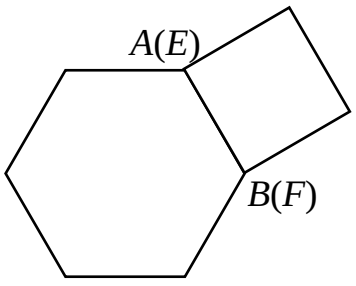
（A）5　（B）6　（C）7　（D）8　（E）9

【Solution】

Since the remainder is 24, the required two-digit numbers must be greater than 24. Observe that $304=280+24$, so we factorize 280 into prime factors: $280=2^3\times 5\times 7$. Thus the required two-digit numbers are $2^2\times 7=28$, $5\times 7=35$, $2^3\times 5=40$, $2^3\times 7=56$ and $2\times 5\times 7=70$. There are 5 such two-digit numbers.

Answer：（A）

16. On the table is a regular hexagon and a square. The side AB of the hexagon coincides with the side EF of the square. With the hexagon fixed, the square rotates about a common vertex until another side of the hexagon coincides with another side of the square. How many such rotations will it take to bring EF back to AB again?



（A）20　（B）18　（C）12

（D）10　（E）6

【Solution】

In order to bring EF back to AB again, the number of rotations must be a common multiple of the number of sides of a regular hexagon and the number of sides of a square. Since the least common multiple of 6 and 4 is 12, we need to rotate at least 12 times to bring EF back to AB again.

Answer：（C）

17. In a standard clock, the angle between two of its hands is the angle they form which is 180° or less. In which of the following five times will the angle between the minute and second hands be greater than or equal to the angle between the hour and the second hand?

（A）06：00：15　　（B）10：10：30　　（C）12：30：18

（D）14：50：00　　（E）20：20：00

【Solution】

Since 1 hour = 60 minutes = 360 seconds, the second hand turns $6°$ per second, the minute hand turns $0.1°$ per second and the hour hand turn $\frac{1°}{120}$ per second.

In (A)： At 06：00：00, the hour hand points to 6, the minute and second hands point to 12. After 15 seconds, the second hand points to 3, the hour hand points to the right of 12 and the minute hand points to the left of 6. But the minute hand turns faster than the hour hand, so the angle between the minute and second hands is less than the angle between the hour and the second hand.

In (B)： At 10：10：00, the hour hand points to a point between 10 and 11, the minute hand points to 2 and the second hand point to 12. After 30 seconds, the second hand points to 6, the minute hand points to the right of 2 and the hour hand still points to a point between 10 and 11. So the angle between the minute and second hands is less than the angle between the hour and the second hand.

In (C)： At 12：30：00, the hour hand points to the middle of 12 and 1, the minute hand points to 6 and the second hand points to 12. After 18 seconds, the angle between the minute and second hands is $180° + 0.1° \times 18 - 6° \times 18 = 73.8°$ and the angle between the hour and the second hand is $6° \times 18 - 15° - \frac{1°}{120} \times 18 = 92.85°$. So the angle between the minute and second hands is less than the angle between the hour and the second hand.

In (D)： At 15：00：00, the hour hand points to 3, the minute and second hands point to 12. 10 minutes ago, the minute hand points to 10, the hour hand points to a point between 2 and 3 and the second hand still points to 12. So the angle between the minute and second hands is less than the angle between the hour and the second hands.

In (E)： At 20：20：00, the hour hand points to a point between 8 and 9, the minute points to 4 and the second hands point to 12. So the angle between the minute and second hands is greater than the angle between the hour and the second hand.

Answer：（E）

18. A sack of 5 kg of rice costs 48 dollars. A sack of 10 kg costs 92 and a sack of 25 kg costs 210 dollars. If we want the average cost per kg of rice to be 9 dollars, how many sacks of rice do we have to buy?

（A）4　　（B）5　　（C）6　　（D）7　　（E）8

【Solution】

For a sack of 5 kg of rice, the price per kg is $48 \div 5 = 9.6$ dollars. For a sack of 10 kg of rice, the price per kg is $92 \div 10 = 9.2$ dollars. For a sack of 25 kg of rice, the price per kg is $210 \div 25 = 8.4$ dollars. The prices per kg for a sack of 5 kg of rice and for a sack of 10 kg of rice are both greater than 9 dollars, so we need to buy at least one sack of 25 kg of rice.

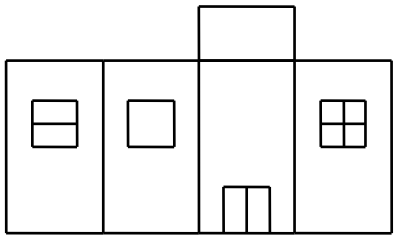
If we buy exactly one sack of 25 kg of rice, the total price is $25 \times (9 - 8.4) = 15$ dollars less than the total price such that the average cost per kg of rice is 9 dollars. Since the price of a sack of 5 kg of rice and the price of a sack of 10 kg of rice are respective $48 - 9 \times 5 = 3$ dollars and $92 - 9 \times 10 = 2$ dollars more than the total price such that the average cost per kg of rice is 9 dollars, we need to buy as many sacks of 5 kg of rice as possible so that the total number of sacks is as less as possible. Observe that $15 \div 3 = 5$, so we need to buy 5 sacks of 5 kg of rice so that the average cost per kg of rice is 9 dollars. Thus we need to buy at least $5 + 1 = 6$ sacks of rice.

If we buy at least two sacks of 25 kg of rice, the total price is at least $50 \times (9 - 8.4) = 30$ dollars less than the total price such that the average cost per kg of rice is 9 dollars. Since the price of a sack of 5 kg of rice and the price of a sack of 10 kg of rice are respective $48 - 9 \times 5 = 3$ dollars and $92 - 9 \times 10 = 2$ dollars more than the total price such that the average cost per kg of rice is 9 dollars, we need to buy as many sacks of 5 kg of rice as possible so that the total number of sacks is as less as possible. Observe that $30 \div 3 = 10$, so we need to buy at least 10 sacks of 5 kg of rice so that the average cost per kg of rice to be 9 dollars. Thus we need to buy at least $10 + 2 = 12$ sacks of rice.

Thus we have to buy 6 sacks of rice.

Answer：（C）

19. How many different rectangles (including squares) in different positions are there in the diagram below?



（A）25　　（B）26　　（C）27

（D）28　　（E）29

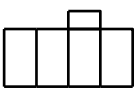
【Solution】

In the diagram, there are four kinds of rectangles:

(i) those formed by one rectangle (including square): there are 14 such rectangles in different positions.

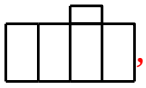
(ii) those formed by two rectangles (including square):

in  , there is one such rectangle; in  , there is another such rectangle;

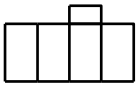
in  , there are 4 such rectangles; in  , there are 4 such rectangles;

so there are 10 rectangles in different positions.

(iii) thopse formed by three rectangles (including square):

in  , there are 2 such rectangles.

(iv) those formed by four rectangles (including square):

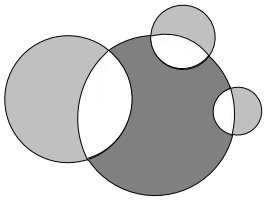
in  , there is 1 such rectangle; in  , there is 1 such rectangle;

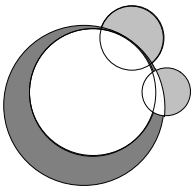
so there are 2 rectangles in different positions.

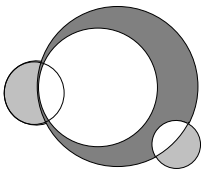
Thus there are totally $14+10+2+2=28$ rectangles in different positions.

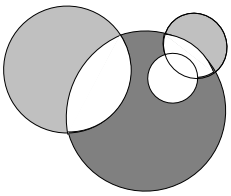
Answer：（D）

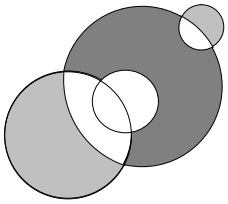
20. In each of the five diagrams, there are four circles with respective radii 7, 6, 3 and 2 cm. For which diagram is the area of the non-overlapping part of the largest circle equal to the total area of the non-overlapping parts of the other three circles?

（A） 

（B） 

（C） 

（D） 

（E） 

【Solution】

Observe that the radii of the four circles are 7cm, 6cm, 3cm and 2cm and that $7^2=6^2+3^2+2^2$. Thus the area of the largest circle is equal to the sum of the areas of the other three circles.

If two circles overlap, the areas of the overlapping region in each circle are equal. So when the three smaller circles overlap with the largest circle with no overlapping parts between any two of the three smaller circles, the area of the overlapping part of the largest circle is equal to the total area of the overlapping parts of the other three circles.

If there are no overlapping parts between any two of the three smaller circles, then the area of the non-overlapping part of the largest circle is equal to the area of the largest circle minus the overlapping part of the largest circle. The total area of the non-overlapping parts of the other three circles is equal to the total area of the other three circles minus the overlapping part of the largest circle. In order to make the area of the non-overlapping part of the largest circle equal to the total area of the non-overlapping parts of the other three circles, we need to make the area of the largest circle equal to the total area of the other three circles and there are no overlapping parts between any two of the three smaller circles.

Hence only (A) satisfies the conditions because there are overlapping parts between the smaller circles in the other options.

Answer：（A）

21. Every student in a class is either in the mathematics club or the language club, and one third of them are in both. If there are 22 students in the language club, 4 less than the number of students in the mathematics club, how many students are there in this class?

【Solution】

From the conditions, there are $22+4=26$ students in the mathematics club. Suppose there are x students in the both clubs, then there are $3x$ students in this class. So $22+26-x=3x$. Thus $x=12$ and hence there are $12\times 3=36$ students in this class.

Answer：036

22. The numbers 1 to 9 form a 3 by 3 table. The sum of every pair of adjacent numbers along a row or a column is computed. What is the largest total of these sums?

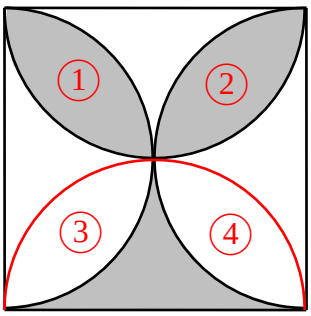
A	B	C
D	E	F
G	H	I

【Solution】

Mark A, B, C, D, E, F, G, H and I to represent the numbers in the squares as shown in the figure. From the conditions, when we add all of the sums, A, C, G and I are added twice, B, D, F and H are added three times and E is added four times. To find the largest total, E should be 9 and A, C, G and I should be 1, 2, 3 and 4. So the largest total of these sums is $(1+2+3+4)\times 2+(5+6+7+8)\times 3+9\times 4=134$.

Answer：134

23. The diagram below shows a square of side length 20 cm, with three semicircle drawn inside it, with three of its sides as diameters. What is the area, in cm^2, of the shaded region?（Take $\pi=3.14$）



【Solution】

Draw a semicircle with the fourth side of the square as diameter, as shown in the figure. Observe that the areas of the regions marked by ①, ②, ③ and ④ are all equal, so we can move the regions ① and ② to regions ③ and ④.

Thus the area of shaded region is equal to the area of a semicircle with a side of the square as diameter, which is $\frac{1}{2}\pi(\frac{20}{2})^2=\frac{1}{2}\times 3.14\times 100=157\ \text{cm}^2$.

Answer：157

24. The International Article Number has 13 digits *ABCDEFGHIJKLM*. Here *M* is a check digit. Let $S = A + 3B + C + 3D + E + 3F + G + 3H + I + 3J + K + 3L$. If *S* is a multiple of 10, then *M* is chosen to be 0. Otherwise it is chosen to be $M = 10 - t$ where *t* is the remainder obtained when *S* is divided by 10. The Code for a certain Article Number is 6901020□09017. What is the missing digit?

【Solution】

From the conditions, we have

$$S = 6+3\times9+0+3\times1+0+3\times2+0+3\times\square+0+3\times9+0+3\times1 = 72+3\times\square.$$

Since $M = 7$, $10-7=3$ is the remainder obtained when $72+3\times\square$ is divided by 10. Thus the units digit of $3\times\square$ is 1. So $\square = 7$.

Answer：007

【Note】

The International Article Number is a code so that

$A + 3B + C + 3D + E + 3F + G + 3H + I + 3J + K + 3L + M$ is divisible by 10.

25. When a three-digit number is increased by 1, the sum is divisible by 15. When it is decreased by 3, the difference is divisible by 8. The sum of it and the number obtained from it by reversing the order of the digits is divisible by 10. What is this number?

【Solution】

Suppose the three-digit number is $\overline{abc}$.

Since $\overline{abc}$ plus 1 is a multiple of 15, it is also a multiple of 5. So $c = 4$ or 9.

Since $\overline{abc}$ minus 3 is a multiple of 8, $\overline{abc}$ is odd. So $c \neq 4$ and hence $c = 9$.

Since $\overline{abc} + \overline{cba}$ is a multiple of 10, $a + c = 10$ and hence $a = 1$.

Thus the three-digit number is $\overline{1b9}$. $\overline{1b9}$ plus 1 is a multiple of 15, so it is also a multiple of 3. Thus $b = 1$, 4 or 7.

Since $\overline{1b9} = 8m + 3$, $\overline{1b6} = 8m$. But $b = 1$ or 4 cannot satisfy the given condition. So the three-digit number is 179.

Answer：179