Solution to
Sixth International Mathematics Assessment for Schools
Round 1 of Upper Division

1. What is the simplified value of $\dfrac{20\times 17}{2+0+1+7}$?

(A) 340　　(B) $\dfrac{34}{2017}$　　(C) 10　　(D) 20　　(E) 34

【Solution】

$\dfrac{20\times 17}{2+0+1+7}=\dfrac{20\times 17}{10}=2\times 17=34$.

Answer: (E)

2. What is the remainder when 2017 is divided by 9?

(A) 0　　(B) 1　　(C) 2　　(D) 3　　(E) 7

【Solution 1】

$2017=9\times 224+1$.

【Solution 2】

The remainder of an integer divided by 9 equals to the remainder of the sum of its digits divided by 9. Thus, the remainder of 2017 is $2+0+1+7=10$ divided by 9, which is 1.

Answer: (B)

3. Positive integers are arranged in the array as shown below, what is the sum of all the integers located on the fifth row ?

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & 3 & 4 & & \\ & 5 & 6 & 7 & 8 & 9 & \\ 10 & 11 & 12 & 13 & 14 & 15 & 16 \\ & & & \vdots & & & \end{array}$$

(A) 91　　(B) 164　　(C) 172　　(D) 189　　(E) 215

【Solution】

According to the pattern, the sum of the fifth row is

$$17+18+19+20+21+22+23+24+25=189.$$

Answer: (D)

4. Arrange the numbers $2.\overline{718}$, $2.7\overline{18}$, $2.71\overline{8}$ and 2.71828 in increasing order. (Repeating decimals are denoted by drawing a horizontal bar above the recurring figures.)

(A) $2.7\overline{18}<2.\overline{718}<2.71828<2.71\overline{8}$　　(B) $2.71828<2.7\overline{18}<2.\overline{718}<2.71\overline{8}$

(C) $2.7\overline{18}<2.71828<2.\overline{718}<2.71\overline{8}$　　(D) $2.71828<2.71\overline{8}<2.7\overline{18}<2.\overline{718}$

(E) $2.7\overline{18}<2.\overline{718}<2.71\overline{8}<2.71828$

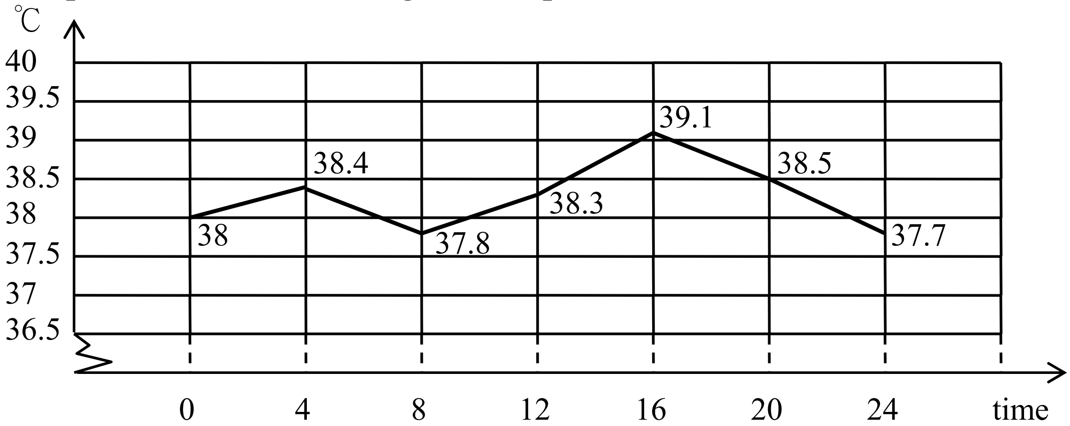
【Solution】

Observe that $2.\overline{718}=2.718718\cdots$, $2.7\overline{18}=2.71818\cdots$, $2.71\overline{8}=2.71888\cdots$ and 2.71828. By comparing the ten thousandths digit, we get

$$2.7\overline{18}<2.71828<2.\overline{718}<2.71\overline{8}.$$

Answer: （C）

5. The figure below plots the body temperature records of one patient in a day. The records started at 00:00 AM and were taken every 4 hours. After how many hours did the patient recorded his highest temperature?



（A）0　（B）4　（C）12　（D）16　（E）24

【Solution】

Reading from the plot, the temperature is the highest at 16 o'clock.

Answer: （D）

6. On a 5×5 table below, place into each cell the sum of its row number and column number. For example, value of a below is $2+3=5$. How many odd numbers are filled into the table?

	1	2	3	4	5
1					
2			a		
3					
4					
5					

（A）5　（B）10　（C）12　（D）18　（E）25

【Solution 1】

All numbers filled in are as follows, there are 12 odd numbers.

	1	2	3	4	5
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

【Solution 2】

Observe that the properties of pari are as follows: odd + odd = even, even + odd = odd and even + even = even, there are 2 odds on the first row, 3 on the second row, 2 on the third row, 3 on the fourth row and 2 on the fifth row. Thus, there is a total of $2+3+2+3+2=12$ odd numbers.

Answer: （C）

7. There are 23 kids seated in a row. They call out the numbers from left to right as 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, … for the first round. They call out 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, … from right to left for the second round. How many kids call out the same number in two rounds?

（A） 11　　（B） 12　　（C） 15　　（D） 18　　（E） 23

【Solution】

23 kids call out the numbers as follows:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
First Call	1	**2**	3	**4**	1	**2**	3	**4**	1	**2**	3	**4**	1	**2**	3	**4**	1	**2**	3	**4**	1	**2**	3
Second Call	3	**2**	1	**4**	3	**2**	1	**4**	3	**2**	1	**4**	3	**2**	1	**4**	3	**2**	1	**4**	3	**2**	1

11 kids call out the same number in two rounds.

Answer: （A）

8. Salted water with 3.2% concentration weights 500 g. How many salt, in grams, is left if the water is vaporized completely?

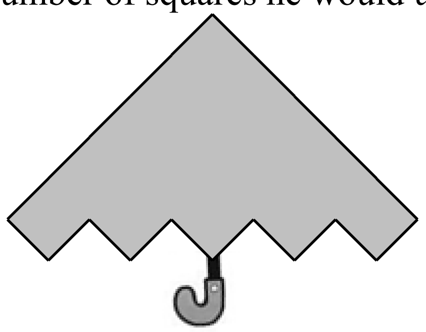
（A） 16　　（B） 32　　（C） 64　　（D） 100　　（E） 128

【Solution】

The salt is $500\times3.2\%=16$ g.

Answer: （A）

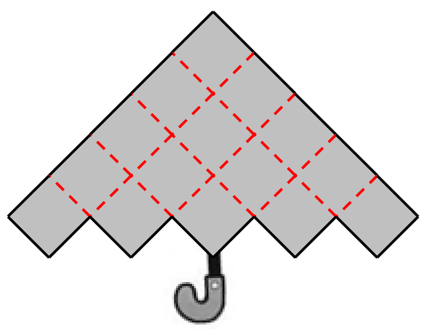
9. In the figure below, Tom combined some squares of the same size into a shape of umbrella. Find the least number of squares he would use.



（A） 5　　（B） 9　　（C） 12　　（D） 15　　（E） 20

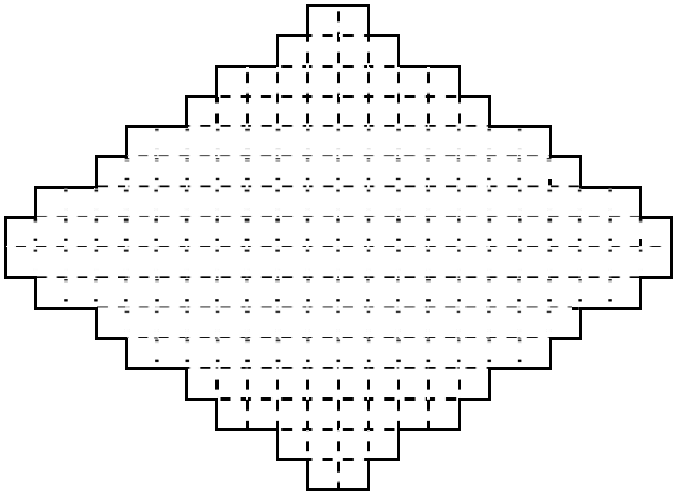
【Solution】

The larger the square is, the less number Tom needs. The square size is determined by the shortest length of the broken line segment enclosing the umbrella. As in the figure, the least number is using 15 squares.



Answer: （D）

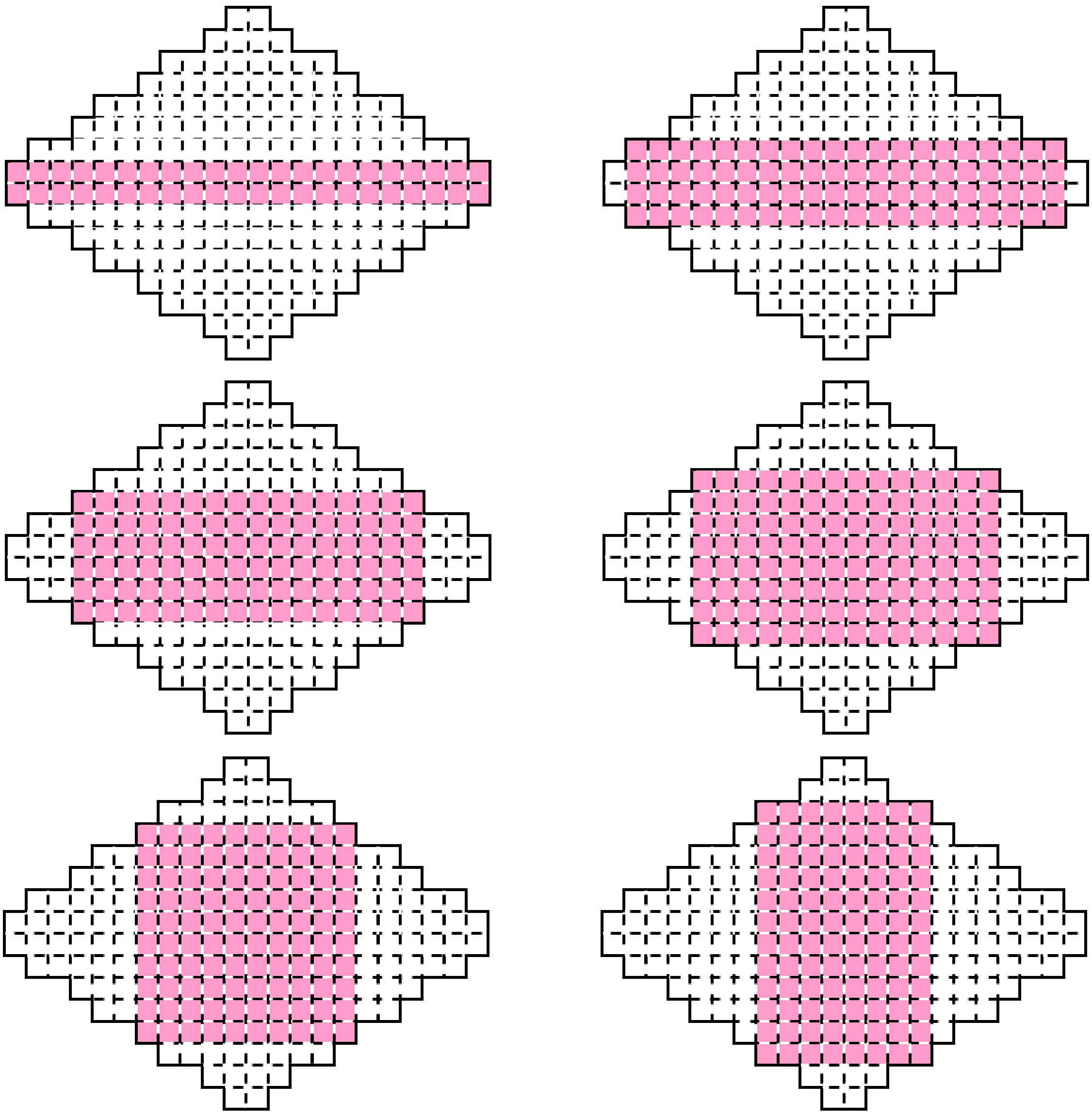
10. The shape enclosed by solid lines in the figure below is composed of unit squares. What is the maximum area of a rectangle that can be cut from the shape along grid lines?

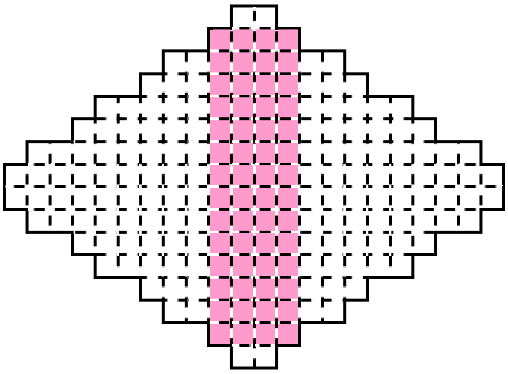


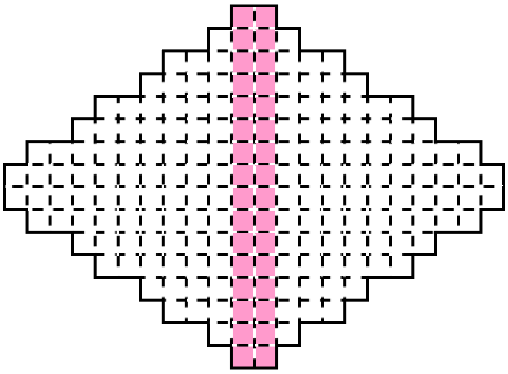
(A) 80 (B) 96 (C) 100 (D) 112 (E) 128

【Solution】

Compute the area of rectangles of following shapes: $2\times22=44$, $4\times20=80$, $6\times16=96$, $8\times14=112$, $10\times10=100$, $12\times8=96$, $14\times4=56$ and $16\times2=32$. The maximal area is 112.







Answer: (D)

11. Given six cards with numbers 1, 2, 3, 4, 5 and 6 one card for each number. Each time Lee takes 2 cards, he computes the difference (larger one minus small one) and discards the two cards. Find the maximum possible sum of the three differences after all cards are discarded.

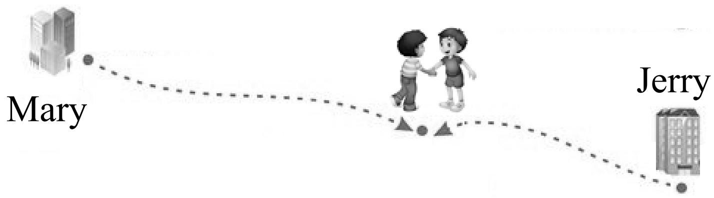
(A) 3 (B) 5 (C) 7 (D) 8 (E) 9

【Solution】

The sum of the differences is the sum of three of the six numbers minus the sum of the remaining three. The maximum is $6+5+4-3-2-1=9$.

Answer: (E)

12. The houses of Mary and Jerry are connected by a trail. One day, they started from their respective house at the same time, and walked towards the other's house. The speed of Mary is 1.5 times that of Jerry and they met 12 minutes later. On the next day, Mary left his house and walked to Jerry's house with the same speed. How long would he take to reach Jerry's house?



(A) 15 (B) 18 (C) 20 (D) 24 (E) 30

【Solution 1】

Since Mary's speed is 1.5 times of Jerry. It will take Mary $12\div1.5=8$ minutes from the place they met the first day to Jerry's house. Mary will take $12+8=20$ minutes to reach Jerry's house.

【Solution 2】

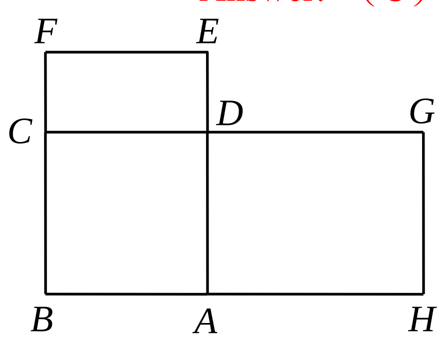
Assume the speed of Jerry is 1, then the speed of Mary is 1.5. The distance between two houses is $(1+1.5)\times12=30$. It will take Mary $30\div1.5=20$ minutes to walk.

Answer: (C)

13. In the figure below, the area of square *ABCD* is 36 cm^2, the area of rectangle *CDEF* is 18 cm^2 and the area of *ADGH* is 48 cm^2. What is the perimeter, in cm, of *BFEDGH*?

(A) 18 (B) 36 (C) 46

(D) 48 (E) 56



【Solution】

The area of square *ABCD* being 36 cm^2, its side length is $AB = BC = 6$ cm. Rectangle *CDEF* has area 18 cm^2, one half of area of *ABCD*, so $CF = \frac{1}{2}BC = 3$ cm; Rectangle *ADGH* has area 48 cm^2, $\frac{4}{3}$ of area of *ABCD*, $CF = \frac{4}{3}AB = 8$ cm. Thus, the perimeter of *BFEDGH* is $(6+3+6+8)\times 2 = 46$ cm.

Answer: (C)

14. There are two routes starting in a bus stop. A bus departs for the first route every 8 minutes and departs the second route every 10 minutes. At 6:00 in the morning, two buses depart for the two routes at the same time. Among the choices below, when will the buses depart for the two routes simultaneously?

(A) 7:30 (B) 8:20 (C) 9:40 (D) 10:00 (E) 11:00

【Solution】

Every $[8,10]=40$ minutes the two buses depart at the same time. Among the options, only the difference between 10:00 and 6:00 is an integer multiple of 40 minutes.

Answer: (D)

15. Let *a*, *b*, *c*, *d*, *e* and *f* are distinct digits such that the expression $\overline{ab}+\overline{cd}=\overline{ef}$. What is the least possible value of $\overline{ef}$?

(A) 30 (B) 34 (C) 36 (D) 39 (E) 41

【Solution】

Since $a+c \geq 1+2=3$, one has $e \geq 3$. Take $e=3$, one gets *a* and *c* are 1 and 2, respectively. Moreover *b* and *d* are non-zero, otherwise *f* is equal to *b* or *d*. So $b+d \geq 4+5=9$, that is, $f=9$. For example, $14+25=39$ and $15+24=39$.

Answer: (D)

16. Henry starts working at 9∶00 in the morning and finishes at 5∶00 in the afternoon. How many more degrees does the minute hand rotates than the hour hand does on the clock during this period?

(A) 120 (B) 1200 (C) 1320 (D) 2640 (E) 2880

【Solution】

Henry works for 8 hours. In this period, the minute hand rotates 8 rounds, that is $8\times 360=2880$ degrees. And the hour hand rotates $\frac{8}{12}=\frac{2}{3}$ round, that is $\frac{2}{3}\times 360=240$ degrees. The difference is $2880-240=2640$ degrees.

Answer: (D)

17. The average score of a class in an exam is 70. Two students got 0 for absence and the average of remaining students is 74. What is the total number of students in the class?

(A) 25 (B) 28 (C) 30 (D) 35 (E) 37

【Solution 1】

The average changes from 74 to 70 when two 0 score students are counted. Every other student gives 4 points of himself to the two zero score student to get a total $70\times 2=140$. There are $140\div 4=35$ other student and $35+2=37$ students in total.

【Solution 2】

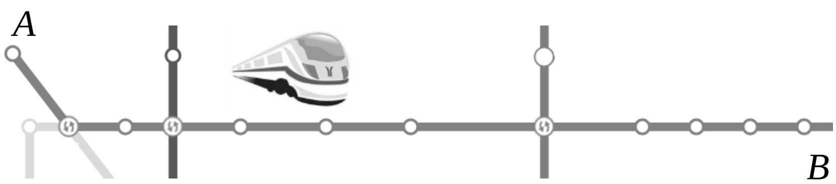
Assume there are x students in the class, then

$$74(x-2)=70x$$

which solves $x=37$.

Answer: （E）

18. The price criteria of the subway ticket of a city is as follows: \$2 for within 4 km, \$1 more per 4 km for distances between 4 km and 12 km, \$1 more per 6 km for distances over 12 km. It costs \$8 to take subway from station A to station B. Which of the following is closest to the distance between A and B?



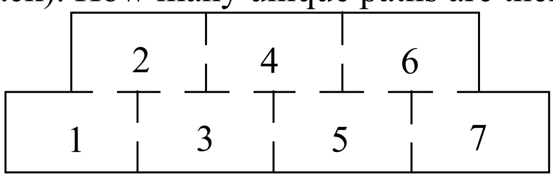
（A）12 km （B）18 km （C）24 km （D）36 km （E）48 km

【Solution】

The cost of a 12km trip is $2+\dfrac{(12-4)}{4}=4$ dollars, so the distance between A and B is over 12 km, and the cost increases by \$4 after 12 km. So the distance after 12 km is at least $6\times 4=24$ km but less than $6\times 5=30$ km. Then the distance between station A and station B is at least 36 km, and at most 42 km. Among the options, the closest distance between A and B is 36 km.

Answer: （D）

19. The figure below shows the floor plan of a library. Each room is connected to adjacent rooms. One starts from room 1 and walks through all rooms without repetitions (going back). How many unique paths are there?



（A）4 （B）8 （C）10 （D）12 （E）13

【Solution】

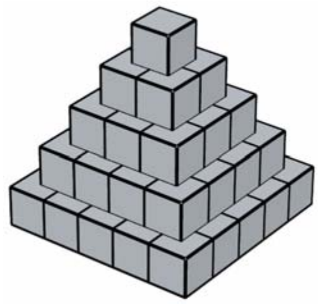
We have all paths by enumeration:

$1\to2\to3\to4\to5\to6\to7$; $1\to2\to3\to4\to5\to7\to6$; $1\to2\to3\to4\to6\to5\to7$;
$1\to2\to3\to4\to6\to7\to5$; $1\to2\to3\to5\to4\to6\to7$; $1\to2\to3\to5\to7\to6\to4$;
$1\to2\to4\to3\to5\to6\to7$; $1\to2\to4\to6\to7\to5\to3$; $1\to3\to2\to4\to5\to6\to7$;
$1\to3\to2\to4\to5\to7\to6$; $1\to3\to2\to4\to6\to5\to7$; $1\to3\to2\to4\to6\to7\to5$;
$1\to3\to5\to7\to6\to4\to2$.

There are totally 13 paths.

Answer: （E）

20. In the figure below, 55 unit cubes were stacked in a pile. Paint the surface of the pile of cubes but the face on the ground is not painted. How many unpainted unit cubes are there when the stack is separated?



(A) 6 (B) 9 (C) 13

(D) 14 (E) 18

【Solution】

The level on the ground has $3\times3=9$ unpainted cubes, the second level has $2\times2=4$, the third level has 1. In total, there are $9+4+1=14$ unpainted unit cubes.

Answer: (D)

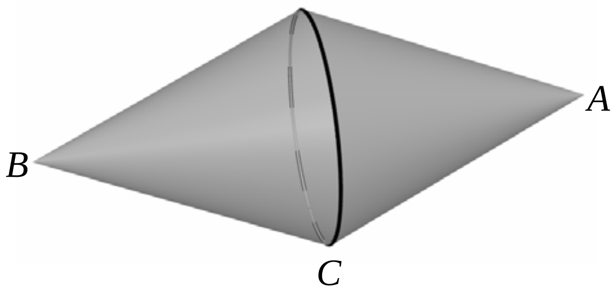
21. There are 50 mail boxes. One day 151 letters are distributed into these mail boxes. It turns out that one mail box has more letters than any other mail box. What is minimal number of letters this mail box can have?

【Solution】

Since $151=3\times50+1$, at least one mail box has at least $3+1=4$ letters. When one mail box has 4 letters, other mail box has 3 letters, the conditions in the problem is satisfied. The minimum is 4.

Answer: 004

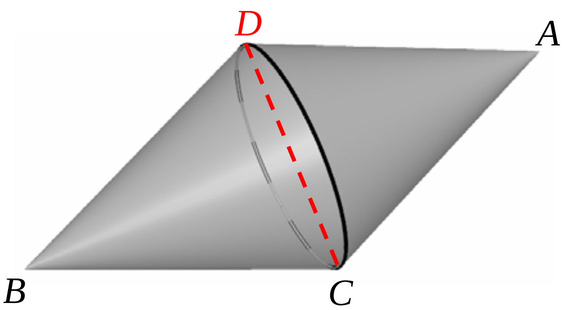
22. In the figure below, a bottle consists of side faces of two identical cones. The radius of the bottom of the cone is 5 cm while the distance between *A* and *B* is 24 cm. There is a hole at *A* and is sealed elsewhere. The bottle is now filled up with water and placed on a horizontal table with *BC* along the table top face. What is the volume, in cm^3, of the water left, if the thickness of the bottle surface and size of hole is ignored (take π as 3.14)?



【Solution】

Assume the diameter of bottom of the cone is *CD*. If *BC* is horizontal, it follows that *AD* is also horizontal. There are no water leaking out. The volume of water is equal to the volume of the bottle, which is

$(\frac{1}{3}\pi\times5^2\times\frac{24}{2})\times2\approx628\,\text{cm}^3$.



Answer: 628

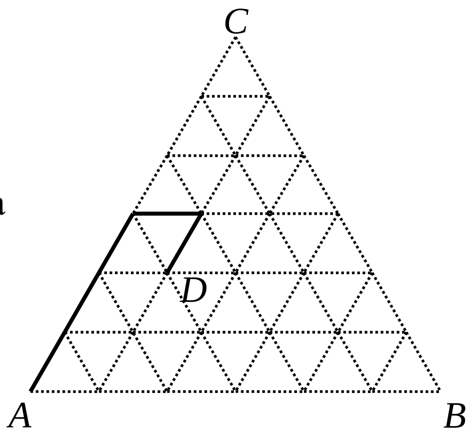
23. The sum of all the digits of a 3-digit number is divided by 4. Now, the sum of all digits of the number which is equal to the previous 3-digit number plus 1 is also divided by 4. What is the largest possible such number?

【Solution】

If there is no carry when the number is added by 1, sum of digits increases by 1, contradiction to that both sum of digits are divided by 4. If there is a carry, the last digit is 9. Since the sum of all digits changes by 8 when one carry happens and by 17 when two carries happens. It follows that only one carry should happen and the original number has the sum of all digits divided by 4. In order to take the number to be maximal, the first digit should also be 9. The second digit can be either 2 or 6. The largest one is 969.

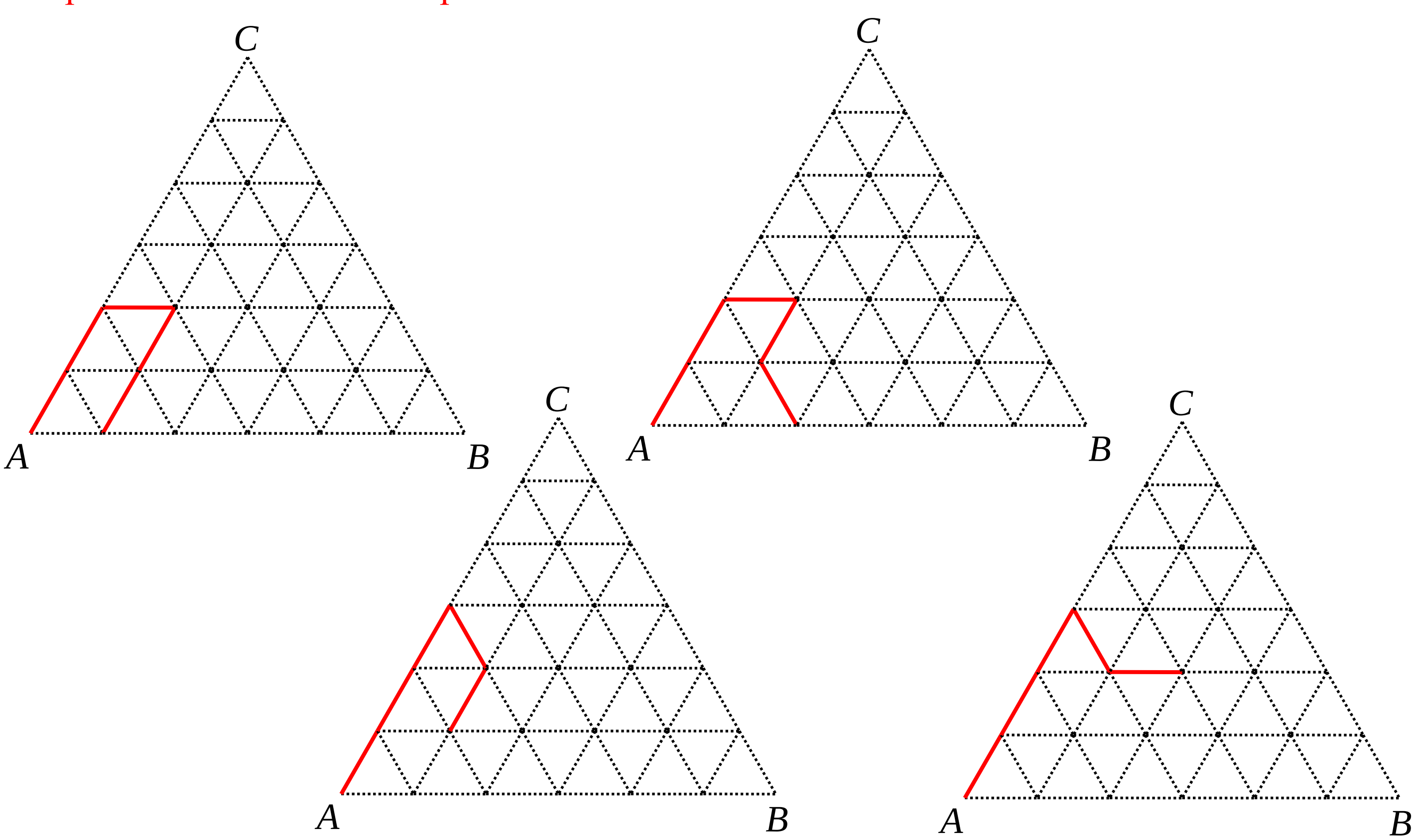
Answer: 969

24. In the figure below, the side length of equilateral triangle *ABC* is 6 cm. Each side is divided into 6 equal segments and connects corresponding dividing points to get an equilateral network. Call a point "reachable" if it can be connected to *A* by a broken line of length 5 cm along the grid lines without passing any lattice point twice. For example, point *D* in the figure is reachable. Find the number of reachable points in the figure.



【Solution】

Let us name those connecting to *A* by a grid line segment of length 1 cm as "one step". From the figure, we know it needs at least 6 steps to reach a point on line *BC*. Then points on *BC* are not reachable. Actually, it is easy to discover that any other point is reachable. Some paths are as follows:



So totally there are $2+3+4+5+6=20$ reachable lattice points.

Answer: 020

25. The students in a research class are clustered into two groups: the morning and afternoon sessions. A student takes part in exactly one group in each session (the two groups in each session can be different and the number of students in each group can be different). Each group has at least one student and at most 6 students. Each student reports the number of students in the group he or she belongs to in two sessions. One finds that no two students report the same pair of numbers (with order, for example, (1, 4) and (4, 1) are different). What is maximum number of students in the class?

【Solution】

Consider the two numbers reported by a student as an ordered pair. The first number represents number of students in the morning session, the second number represents the afternoon session. Since maximum number of students in a group is 6, there is a total of 36 combinations as follows:

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Since the pairs in row k ($1 \le k \le 6$) corresponds students in a group of size k in the morning, the total number of students in row k is a multiple of k. There are at most 6 students in row 4, 5 students in row 5. Similarly, the number of students in column k is a multiple of k. If (4, 4), (4, 5), (5, 4) are taken out as without students, the remaining 33 pairs satisfy the above requirement. Group of the students in two sessions row-wisely and column-wisely respectively, one gets a feasible grouping. So the maximum number of students is 33.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)	
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)	
(4, 1)	(4, 2)	(4, 3)	~~(4, 4)~~	~~(4, 5)~~	(4, 6)	→ Must remove 2
(5, 1)	(5, 2)	(5, 3)	~~(5, 4)~~	(5, 5)	(5, 6)	→ Must remove 1
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)	
			↓ Must remove 2	↓ Must remove 1		

Answer: 033